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# ASSIGNMENT - 1
## ENGINEERING PHYSICS


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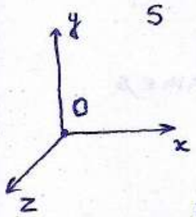
Q1. What do you mean by frame of references ? Differentiate b/w inertial & non inertial frame of references.

### FRAME OF REFERENCE




For making observations about motion, one has to fix the position of observer.
We then assume a set of Axes to which observer is permanently attached. we call such set of Axes as frame of reference

### REAL FORCES

In classical mechanics, real forces occur in PAIRS and result from interaction of two bodies and their magnitude decays as the distance b/w interacting bodies increase.

An isolated object ($\infty$ distant) is thus expected not to have any real force acting on it. Hence it should be having a CONST. VELOCITY.

Galileo and Newton emphasized that it is constant velocity, not rest which is the NATURAL STATE OF MOTION, when there is no real force acting on a body.

### INERTIAL FRAME OF REFERENCE

A frame of reference in which an isolated object (ie which experiences no real force) is found to move at const. velocity (same magnitude & direction) is called inertial frame of reference.

- There can be $\infty$ number of inertial frames, but relative velocity b/w them must be constant.
- All inertial frame of references will measure the velocity of an isolated object constant wrt their time, however, the magnitude of velocity measured may not be same.

## NON INERTIAL FRAME OF REFERENCE

It is a frame of reference accelerating wrt an inertial frame of reference. In such a frame law of inertia does not hold ~~and all physical laws become variant~~ and Newton's laws of motion are non consistent.

Q2. State Postulates of special theory of relativity. Do Galilean transformations satisfy them?

EINSTEINS POSTULATES OF SPL. THEORY OF RELATIVITY

1. The laws of Physics are same in all inertial frame of references. No prefered frame exists.
2. The speed of light is same in all inertial frames.

GALILEAN TRANSF. CONFLICTS WITH STR

1. Did not comply with electromagnetic theory which gives speed of light in vacuum as.

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

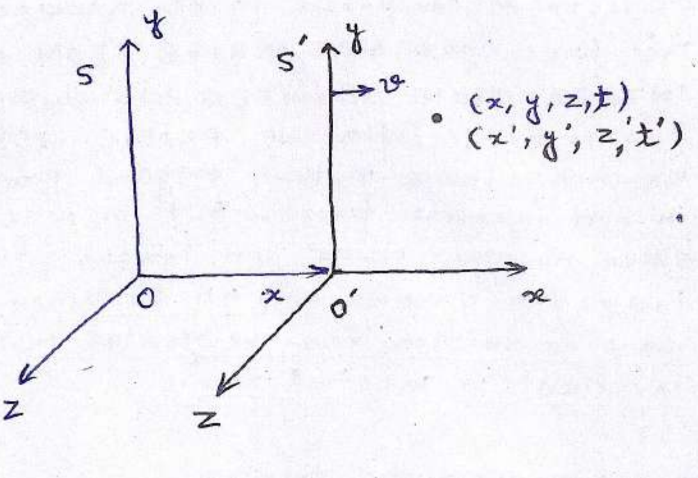
speed of light depends only on FUNDAMENTAL CONSTANTS and it is expected that it is constant in all inertial frames. Galilean approach is opposite of it and predicts ETHER.

Michaelson - Morley experiment itself established its inability to explain electromagnetic theory.

2. SIMULTANEITY of two events is relative under second postulate while is absolute in Galilean transformations.

Hence Galilean transformations do not satisfy postulates of STR

Q3. Write Galilean transformations for ~~constants~~ coordinates. Derive velocity transformations from them.




1D Direct coordinate transformation

$$x' = x - vt$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

inverse transformation

$$x = x' + vt$$
$$y = y'$$
$$z = z'$$
$$t = t'$$

Velocity transformation.
(diff$^n$ coorinate trans. wrt time
$t = t' \rightsquigarrow dt = dt'$ )

$$\frac{dx'}{dt'} = \frac{dx}{dt} - v \rightsquigarrow u_x' = u_x - v$$
$$\frac{dy'}{dt'} = \frac{dy}{dt} \rightsquigarrow u_y' = u_y$$
$$\frac{dz'}{dt'} = \frac{dz}{dt} \rightsquigarrow u_z' = u_z$$

$u_s$ : velocity of particle in S frame
$v$ : relative velocity b/w frames.

inverse velo. transfomation

$$u_x = u_x' + v$$
$$u_y = u_y'$$
$$u_z = u_z'$$

Q4. Show that laws of physics are same in all inertial frames and why they cannot be same in the non inertial frame?

INVARIANCE OF PHYSICAL LAWS

Invariance of Physical laws can be understood seprately in classical physics and physics associated to electromagnetic theory. Also invariance can be identified by comparing to contradictions in non-inertial frames.

Newtons laws in non inertial frames.

Pseudo force

It comes into effect when the frame of reference has acceleration compared to non accelerating frame. This force does NOT ARISE FROM ANY PHYSICAL INTERACTION BETWEEN two objects, but rather due to acceleration of FRAME ITSELF and hence newton's laws in their original form cannot be applied in non inertial frame. But in inertial frame, since no other ~~force~~ non physical force arise, newtons laws are consistent. All the classical physics bearing some derivation from newton's laws is therefore invariant in inertial frame.

Maxwell's electromagnetic theory.

After maxwells theory, it has been established that speed of light is solely dependent on two natural constants and thus it is expected that speed of light shall be constant in all frame of references (inertial). According to classical relativity, speed of light is entitled to variability under relative motion between two inertial frames. Therefore concept of Ether was proposed, which is an absolute frame in which only light of speed is constant and is equal to

$$c = 1/\sqrt{\mu_0 \varepsilon_0}$$

But Einstein emphasized that there cannot exist a preferred frame and thus gave two postulates in special theory of relativity which assured invariancy of classical as well as electromagnetic, and through it quantum laws in inertial frames. The famous Michaelson - Morley experiment proved no existence of preferred frame like ether and thus validated the postulate that all physical laws are invariant in all inertial frame of references.

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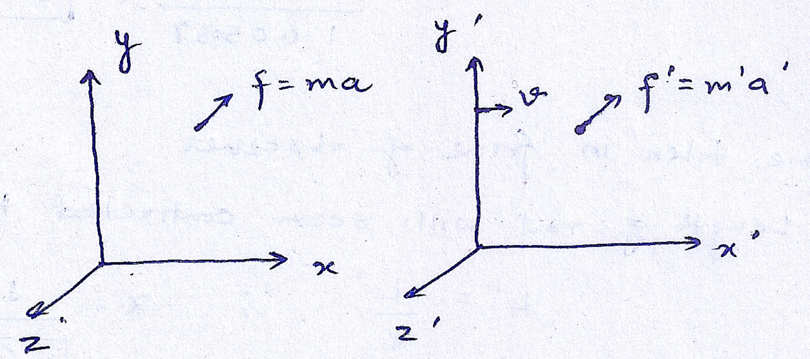
For example, Mathematically we may prove invariance of Newton's second law by transforming it from one inertial frame to other inertial frame.

Say, two inertial frames S & S' exist such that S' is moving with a velocity $v$ relative to S. ~~and~~ An object experiences $f' = m'a'$ force in S' frame. To show invariance of second law, we shall show that it experiences same force $f'$ in S frame as well.

$$f' = m'a'$$
$$= m' \frac{dv'}{dt}$$
$$= m \frac{d}{dt} \frac{d(x - vt)}{dt}$$
$$= m \frac{d}{dt} (v_x - v)$$
$$= m \frac{dv_x}{dt}$$
$$= ma = f \quad \rightsquigarrow f' = f$$




Q5. Show that rest mass of a photon is zero.

Relativistic mass is given by

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

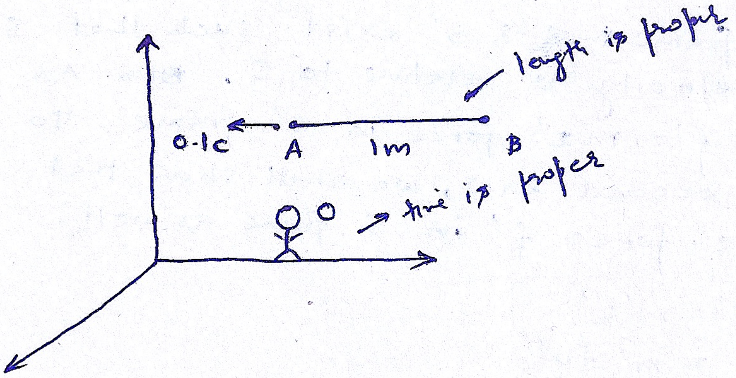
$$\rightsquigarrow \quad m_0 = m\sqrt{1 - v^2/c^2}$$

Since photons travel at speed of light, $v = c$.

$$\rightsquigarrow \quad m_0 = m\sqrt{1 - c^2/c^2} = m\sqrt{1-1} = m\sqrt{0} = 0.$$

Hence, rest mass of photon is zero.

Q6. How much time does a meter stick moving at 0.1c relative to an observer take to pass the observer? The meter stick is parellel to the direction of motion.




(i) Time taken in frame of rod

Proper length of rod is 1m in its own frame because rod is at rest wrt. its own frame
However its time is dilated wrt proper time of observer.

$$t' = \frac{t}{\gamma}$$

$$= \frac{3.316 \times 10^{-8}}{1.005037} = \boxed{3.343 \times 10^{-8} s}$$

(ii) Time taken in frame of observer.

Length of rod will seem contracted to observer

$$L' = \frac{L}{\gamma} \quad ; \quad \gamma = \frac{1}{\sqrt{1-(0.1)^2}}$$

$$= \frac{1}{1.005037} \qquad = 1.005037$$

$$= 0.994987.$$

Time taken by the rod to pass observer

$$t = \frac{0.994987}{0.1 \times 3 \times 10^{8}} \approx \boxed{3.316 \times 10^{-8} s}$$

Q7. A certain quantity of ice at 0°c melts into water at 0°c and in doing so gains 1 kg of mass. What was its initial mass? [Hint: ~~Lanten~~ Latent heat $L = 3.36 \times 10^5$ J/kg]

$m_i$ : initial mass of ice

- Energy gained by ice in melting.

$$Q = m_i L = 3.36 \times 10^5 \frac{J}{\cancel{kg}} \times m_i \cancel{kg} = 3.36 \times 10^5 m_i \ J$$

- Rest energy of ice before melting

$$E_i = m_i c^2 \ J$$

- Rest energy of water after melting [mass increased by 1 kg after melting]

$$E_f = (m_i + 1) c^2 \ J$$

- By conservation of energy,

Energy before = Energy after melting

$$Q + E_i = E_f.$$

$$3.36 \times 10^5 m_i + \cancel{m_i c^2} = (\cancel{m_i} + 1)\cancel{c^2}$$

$$3.36 \times 10^5 m_i = c^2$$

$$m_i = \frac{(3 \times 10^8)^2}{3.36 \times 10^5}$$

$$\boxed{m_i \approx 2.68 \times 10^{11} \text{ kg}}$$

Q8. Calculate relative velocity of two frames, such that length contraction is 10% of the proper length of an object.

Length transformation in relativity is given by

$$L = \gamma L'$$

where, $L'$ : Length measured in moving frame

$L$ : " " in rest frame.

$\gamma$ : Lorentz factor $= 1/\sqrt{1 - v^2/c^2}$

$v$ : Relative velocity b/w frames or velocity of moving frame

ATQ, $\Delta L^{\circ} = 0.1 L \rightsquigarrow L' = L - \Delta L = L - 0.1 L = 0.9 L$

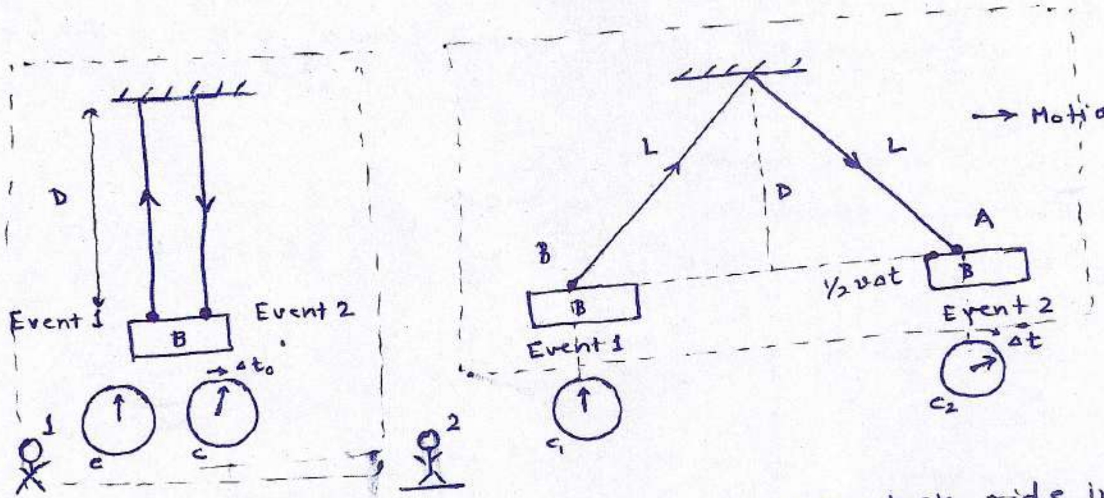
$$\therefore \cancel{L} = \gamma \cancel{L} \times 0.9$$

$$\frac{1}{0.9} = \frac{1}{\sqrt{1 - v^2/c^2}} \rightsquigarrow 1 - \frac{v^2}{c^2} = 0.81$$

$$\frac{v^2}{c^2} = 1 - 0.81 \rightsquigarrow v = \sqrt{0.19}\, c \approx 0.4359\, c$$

$$\therefore \boxed{v \simeq 1.308 \times 10^8 \text{ m/s}}$$

Q9. Derive the formula of time dilation & length contraction.




Observer 1, a light source, a mirror and a clock ride in a train moving with velocity $\bar{v}$ relative to station. A pulse of light leaves light source B (event 1), travels vertically upwards, then reflected downward by mirror and then detected back at source (event 2). Time measured b/w event 1 & 2 is

$$\Delta t_0 = 2D/c \quad \text{(observer 1)}$$

Two events occurred at SAME LOCATION in observer 1's frame and only one clock is required (proper time)

Consider observer 2, who is standing on platform as the train passes and source, mirror and clock are moving in train will observe that two events occur at different LOCATIONS., so to measure the time interval b/w events, two synchronised clocks are needed one at each event. According to 2nd postulate of STR, light travels at same speed for both observers 1 & 2. Time interval measured by obs. 2,

$$\Delta t = \frac{2L}{c} \quad \text{where } L = \sqrt{\left(\tfrac{1}{2} v\Delta t\right)^2 + D^2}$$

$$\therefore \quad L = \sqrt{\left(\tfrac{1}{2} v\Delta t\right)^2 + \left(\tfrac{1}{2} c\Delta t_0\right)^2} \quad - (1)$$

eliminating L from $\Delta t = 2L/c$, eq$^n$ (1) we get

$$\boxed{\Delta t = \Delta t_0 / \sqrt{1-(v/c)^2}} \quad - (2)$$

→ Time dilation

Length measured by observer 2 (of platform), ~~observers pas~~

$L_0 = v\Delta t$ $\qquad L_0$ : proper length because platform is at rest wrt observer 1.

Time taken by observer 2 to cross platform $\Delta t$ : not proper time!

~~However, for observer 1, it passes platform at two diff places~~

According to observer 1, ~~obs 2~~ events measured of obs. 1 occur at same PLACE, to it Length of platform

$$L = v\Delta t_0 \quad \leadsto \quad \frac{L}{L_0} = \frac{v\Delta t_0}{v\Delta t} = \frac{1}{\gamma} \quad \text{(from eq}^n\text{ 2)}$$

→ Length contraction

$$\boxed{L = L_0/\gamma} \quad (L \le L_0 \text{ bec } \gamma \ge 1)$$

Q10. Velocity of light in frame A has components (0.35c, 0.6c, a). In frame B, the components are (0.25c, b, u). Calculate a, b, u. (c is the velocity of light in vacuum)

By second postulate of STR, speed of light is 'c' in all inertial frames.

$\therefore$ for frame A, $\sqrt{(0.35c)^2+(0.60c)^2+a^2} = c$

$$0.1225c^2 + 0.36c^2 + a^2 = c^2$$
$$a^2 = (1-0.4825)c^2$$
$$= 0.5175c^2$$
$$\boxed{a = 0.719c}$$

By relativistic velocity transformation

$$U_B = \frac{U_A - V_{AB}}{1 - \frac{U_A V_{AB}}{c^2}}$$

in x direc$^n$,

$$0.25c = \frac{0.35c - V_{AB}}{1 - \frac{0.35\cancel{c}V_{AB}}{c^{\cancel{2}}}}$$

$$0.25c - 0.35 \times 0.25 V_{AB} = 0.35c - V_{AB}.$$
$$(1-0.0875)V_{AB} = 0.1c.$$
$$V_{AB} \approx 0.1096c$$

in y direc$^n$

$$b = \frac{0.6c - 0.1096c}{1 - \frac{0.6\cancel{c} \times 0.1096\cancel{c}}{\cancel{c^2}}} = \frac{0.4904c}{0.93420}$$

$$\boxed{b \approx 0.525c}$$

Again by second postulate of STR.

for frame B, $\sqrt{(0.25c)^2 + (0.525c)^2 + u^2} = c$

$$0.0625c^2 + 0.2756c^2 + u^2 = c^2$$
$$u^2 = 0.662c^2$$
$$\boxed{u \approx 0.814c^2}$$

Q 11. considering two frame of references O and O', the later moving with a constent velocity $v$ along x-axis. Derive lorentz coordinate and velocity transformations (all compoents), derive Galilean transformations for low velocity range



To maintain homogenity of space, lorentz tranformation is linear in space and time coordinates.

$$x' = B_{xx}x + B_{xy}y + B_{xz}z + B_{xt}t + C_1$$
$$y' = B_{yx}x + B_{yy}y + B_{yz}z + B_{yt}t + C_2$$
$$z' = B_{zx}x + B_{zy}y + B_{zz}z + B_{zt}t + C_3$$
$$t' = B_{tx}x + B_{ty}y + B_{tz}z + B_{tt}t + C_4$$

After taking into account

(i) Special choice of Axes

(ii) Symmetry arguements

several constents can be removed and the equation set is reduced to

$$x' = B_{xx}(x - vt)$$
$$y' = y$$
$$z' = z$$
$$t' = B_{tx}x + B_{tt}t$$

we use second postulate to evaluate remaining constents. Let at time $t = t' = 0$, a spherical light wavefront is emitted from origin. observers in both S & S' will find the spherical wavefront is emerging from their respective centres O and O' with speed c.

$$\therefore \quad x^2 + y^2 + z^2 = c^2t^2 \quad -①$$
$$x'^2 + y'^2 + z'^2 = c^2t'^2 \quad -②$$

substituting $x', y', z', t^2$ from above set of eqⁿs in eqⁿ ②

$$B_{xx}^2(x - vt)^2 + y^2 + z^2 = c^2(B_{tx}x + B_{tt}t)^2$$
$$B_{xx}^2(x^2 + v^2t^2 - 2vxt) + y^2 + z^2 = c^2(B_{tx}^2x^2 + B_{tt}^2t^2 + 2B_{tx}B_{tt}xt)$$
$$(B_{xx}^2 - c^2B_{tx}^2)x^2 + y^2 + z^2 - (B_{xx}^2v + B_{tx}B_{tt}c^2)2xt = (B_{tt}^2c^2 - B_{xx}^2v^2)t^2$$

comparing coefficients with eqⁿ ①

$$B_{xx}^2 - c^2B_{tx}^2 = 1$$
$$B_{tx}B_{tt}c^2 + B_{xx}^2V = 0$$
$$B_{tt}^2c^2 - B_{xx}^2V^2 = c^2$$

Solving the equations, the three unknown coefficients can be determined

$$B_{xx} = B_{tt} = \frac{1}{\sqrt{1 - v^2/c^2}} = \underbrace{\gamma}_{\text{lorentz trasformation}} \quad ; \quad B_{tx} = \frac{-v/c^2}{\sqrt{1 - v^2/c^2}}$$

substituting the constants $B_{xx}$, $B_{tx}$, $B_{tt}$ in the reduced set of eq^ns we get the lorentz transformations as :

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}} \; ; \; y' = y \; ; \; z' = z \; ; \; t' = \frac{t - \frac{vx}{c}}{\sqrt{1 - v^2/c^2}}$$

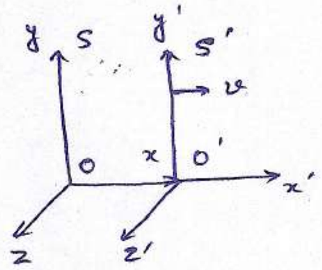
ALTERNATIVE DERIVATION OF LORENTZ TR.

wrt frame S

$$c = \frac{(x^2 + y^2 + z^2)^{1/2}}{t} \rightsquigarrow x^2 + y^2 + z^2 = c^2 t^2 \quad - (1)$$

wrt frame S'

$$c = \frac{(x'^2 + y'^2 + z'^2)^{1/2}}{t} \rightsquigarrow x'^2 + y'^2 + z'^2 = c^2 t'^2 \quad - (2)$$




Transformation between $x$ & $x'$

$$x' = k(x - vt) \quad - (3)$$
$$x = k(x' + vt') \quad - (4)$$
$$= k(k(x - vt) + vt')$$
$$\frac{x}{k} = kx - kvt + vt'$$
$$t' = \frac{x}{vk} - \frac{kx}{v} + kt$$
$$t' = kt - \frac{kx}{v}\left[1 - \frac{1}{k^2}\right] \quad - (5)$$

According to second postulate of STR ⇝ speed of light is same in all inertial frames

$$\therefore \quad x = ct \; ; \; x' = ct \quad . - (6)$$

put (6) in (3), (4)

$$ct' = k(ct - vt) \quad - (7)$$
$$ct = k(ct^2 + vt') \quad - (8)$$

multiply (7) & (8)

$$c^2 \cancel{tt'} = k \cancel{tt'} [c^2 + v^2]$$

$$k^2 = \frac{c^2}{c^2 - v^2} \rightsquigarrow k = \frac{c}{\sqrt{c^2 - v^2}} \rightsquigarrow k = \frac{1}{\sqrt{1 - c^2/v^2}} \quad - (9)$$

$$\boxed{x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}}$$

substituting (9) in eqⁿ (5)

$$t' = kt - \frac{kx}{v}\left[1 - \frac{1}{k^2}\right]$$

$$t' = \frac{1}{\sqrt{1-v^2/c^2}}\left[t - \frac{x}{v}\left[\cancel{1} - \cancel{1} - \frac{v^2}{c^2}\right]\right]$$

$$t' = \frac{1}{\sqrt{1-v^2/c^2}}\left[t - \frac{x}{\cancel{v}}\left[-\frac{v^{\cancel{2}}}{c^2}\right]\right]$$

$$\boxed{t' = \frac{1}{\sqrt{1-v^2/c^2}}\left[t + \frac{xv}{c^2}\right]}$$

Since we had assumed motion of s' wrt s in x-direcⁿ only, y, z remain invariant.

$$\therefore \boxed{\begin{aligned} y' &= y \\ z' &= z \end{aligned}}$$

VELOCITY TRANSFORMATIONS.

$$x' = \frac{x - vt}{\sqrt{1-v^2/c^2}} \quad ; \quad t' = \frac{t - \frac{xv}{c^2}}{\sqrt{1-v^2/c^2}} \quad ; \quad \gamma = \frac{1}{\sqrt{1-v^2/c^2}}$$

Differentiating ~~which~~

$$u_x' = \frac{dx'}{dt'} = \frac{\gamma(dx - v\,dt)}{\gamma\left(dt - \frac{v\,dx}{c^2}\right)} = \frac{\frac{dx}{dt} - v}{1 - \frac{v\frac{dx}{dt}}{c^2}} = \boxed{\frac{u_x - v}{1 - \frac{u_x v}{c^2}}}$$

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$$y' = y \quad ; \quad t' = \gamma\left(t - \frac{vx}{c}\right)$$

Taking differential

$$u_y' = \frac{dy'}{dt'} = \frac{dy}{\gamma\left(dt - \frac{v\,dx}{c^2}\right)} = \frac{dy/dt}{\gamma\left[1 - \frac{v^2}{c^2}\frac{dx}{dt}\right]} = \boxed{\frac{u_y}{\gamma\left[1 - \frac{v^2 u_x}{c^2}\right]}}$$

considering motion along x direcⁿ with rel. vel. $u_x$.

However, if $U = x\hat{i} + y\hat{j} + z\hat{k}$ instead of $U_x = x\hat{i} + 0\hat{j} + z\hat{k}$ u will be there instead of $U_x$.

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similarly for $z' = z$

$$\boxed{u_z' = \frac{u_z}{\gamma\left[1 - \frac{v^2 u_z}{c^2}\right]}}$$

GALILEAN TRANSFORMATION AS LORENTZ TR FOR LOW VELOCITY RANGE

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} \quad ; \quad \beta = \frac{v^2}{c^2}$$

$$\begin{aligned} x' &= \gamma(x - vt) \\ y' &= y \\ z' &= z \\ t' &= \gamma\left[t - \frac{vx}{c^2}\right] \end{aligned} \quad \xrightarrow[\substack{\therefore\ \beta \approx 0 \\ \therefore\ \gamma \approx 1}]{v \ll c} \quad \begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \\ t' &= t - \frac{vx}{c^2} \end{aligned}$$

(lorentz) (Galilean)

Q 12. Obtain the following expressions of relativistic kinetic energy and total energy

$$k = mc^2(\gamma - 1) \text{ and } E = (p^2c^2 + m^2c^4)^{1/2}$$

where $\gamma$ is lorentz factor, $m$ is proper mass.

Relativistic force

$$F = \frac{ma}{(1-v^2/c^2)^{3/2}} = \gamma^3 ma$$

work energy equivelence

$$KE = \int_0^v F\,dx$$

(KE is workdone in accelerating the object from rest to speed $v$)

$$KE = \int_0^v \gamma^3 ma\,dx$$

$$= \int_0^v \gamma^3 m\left(\frac{dv}{dt}\right)dx$$

$$= \int_0^v \gamma^3 m \frac{dx}{dt}\,dv$$

$$= \int_0^v \gamma^3 mv\,dv$$

$$= \int_0^v \frac{mv}{\left(\sqrt{1-v^2/c^2}\right)^3}\,dv$$

Let $u = 1 - v^2/c^2 \rightsquigarrow du = -\frac{2v}{c^2}dv$

$$KE = -\frac{mc^2}{2}\int \frac{du}{u^{3/2}} = -\frac{mc^2}{2}\left[-2u^{-1/2}\right]$$

$$= mc^2\left[\frac{1}{\sqrt{1-v^2/c^2}}\right]_0^v$$

$$= mc^2\left[\frac{1}{\sqrt{1-v^2/c^2}} - 1\right]$$

$$\therefore KE = \underbrace{\gamma mc^2}_{\text{Total energy}} - \underbrace{mc^2}_{\text{rest energy}}$$

$$\boxed{k = mc^2(\gamma - 1)}$$

By mass energy equivalence

$$E = mc^2 = \frac{m_0c^2}{\sqrt{1-v^2/c^2}} \quad ; \quad p = mv = \frac{m_0 v}{\sqrt{1-v^2/c^2}}$$

$$E^2 = \frac{m_0^2c^4}{1-v^2/c^2} \quad ; \quad p^2 = \frac{m_0^2v^2}{1-v^2/c^2}$$

$$E^2 - p^2c^2 = \frac{m_0^2c^4 - m_0^2v^2c^2}{1-v^2/c^2}$$

$$= \frac{m_0^2c^2\,\cancel{(c^2-v^2)}}{\frac{\cancel{c^2-v^2}}{c^2}} = m_0^2c^4$$

$$\therefore E^2 - p^2c^2 = m_0^2c^4.$$

$$E^2 = m_0^2c^4 + p^2c^2$$

$$\boxed{E = \sqrt{m_0^2c^4 + p^2c^2}}$$

Q13 An event happens according to an observer O at (1m, 6m, −3m, $3\times10^{-8}$s). The same event occurs at (1m, 0.75m, −3m, $2.25\times10^{-8}$ s) wrt another observer O'. Find the velocity of O' wrt O (magnitude and direction)

$$U_x = \frac{x}{t} = \frac{1\,m}{3\times10^{-8}s} = \frac{1}{3}\times10^{8}\ m/s$$

$$U_y = \frac{y}{t} = \frac{6m}{3\times10^{-8}s} = 2\times10^{8}\ m/s$$

$$U_z = \frac{z}{t} = \frac{-3}{3\times10^{-8}s} = -1\times10^{8}\ m/s$$

$$U_x' = \frac{x'}{t} = \frac{1\,m}{2.25\times10^{-8}s} = 0.44\times10^{8}\ m/s$$

$$U_y' = \frac{y'}{t} = \frac{0.75\,m}{2.25\times10^{-8}s} = 0.33\times10^{8}\ m/s$$

$$U_z' = \frac{z'}{t} = \frac{-3\,m}{2.25\times10^{-8}s} = -1.33\times10^{8}\ m/s.$$

$$U_{o'o} = U_o - U_{o'} = \left[\frac{1}{3}-(0.44)\right]\times10^{8}\hat{i} + (2-0.33)\times10^{8}\hat{j} + \left[-1+1.33\right]\times10^{8}\hat{k}$$

$$= -0.11\times10^{8}\hat{i} + 1.67\times10^{8}\hat{j} + 0.33\times10^{8}\hat{k}$$

$$= 1.705\times10^{8}\left[\frac{-2}{31}\hat{i} + \frac{334}{341}\hat{j} + \frac{6}{31}\hat{k}\right]$$

Since length contraction is happening along x-axis velocity of O' should be along x-axis of O frame.
According to time transformation,

$$t' = \gamma\left[t - \frac{vx}{c^2}\right]$$

$$2.25\times10^{-8} = \frac{3\times10^{-8} - \frac{v\times6}{3\times10^{8}}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\frac{9}{16}\ 5.0625\times10^{-8}\left[1-\frac{v^2}{c^2}\right] = 9\times10^{-8}\left[1-6v\right]^2$$

$$\frac{9}{16} - \frac{9v^2}{16c^2} = 1 + 36v^2 - 12v$$

$$\left(36+\frac{9}{16}c^2\right)v^2 + 12v + \frac{7}{16} = 0.$$

According to LT, $y' = \gamma(y - vt)$ ; $t' = \gamma(t - vy/c^2)$

$$\frac{y'}{t'} = \frac{\cancel{\gamma}(y - vt)}{\cancel{\gamma}(t - vy/c^2)} \Rightarrow \frac{\cancel{0.75}\ 1}{\cancel{2.25}_{3} \times 10^{-8}} = \frac{6 - v \times 3 \times 10^{-8}}{3 \times 10^{-8} - \frac{v \times 6}{c^2}}$$

$$\frac{1}{3} \times 10^{8} \left[ 3 \times 10^{-8} - \frac{6v}{c^2} \right] = 6 - v \times 3 \times 10^{-8}$$

$$1 - \frac{2 \times 10^{8} v}{c^2} = 6 - 3v \times 10^{-8}$$

$$v \left[ 3 \times 10^{-8} - \frac{2 \times \cancel{10^{8}}}{\cancel{c^2}_{9 \times 10^{16}}} \right] = 6 - 1$$

$$v \left[ 3 \times 10^{-8} - \frac{2}{9 \times 10^{8}} \right] = 6 - 1 \Rightarrow \frac{v}{9 \times 10^{8}} \left[ 27 - 2 \right] = 5$$

$$\Rightarrow v = \frac{5}{25} \times 9 \times 10^{8} = \boxed{1.8 \times 10^{8}\ m/s}$$

Since only y coordinate is variant, velocity is along y-direcn.

$$\therefore \boxed{\vec{v} = +1.8 \times 10^{8}\ \hat{j}\ m/s}$$

Q 14. Show that the relativistic form of Newton's second law, when F is parallel to v is

$$F = m_0 \frac{dv}{dt} \left(1 - \frac{v^2}{c^2}\right)^{-3/2}$$

$$F = \frac{dP}{dt} = \frac{d}{dt} mv$$

$$= \frac{d}{dt} \frac{m_0}{\sqrt{1 - v^2/c^2}} v = m_0 \frac{d}{dt} [v(1 - v^2/c^2)^{-1/2}]$$

$$= m_0 \left[ \frac{dv}{dt} (1 - v^2/c^2)^{-1/2} + v \frac{d}{dt} (1 - v^2/c^2)^{-1/2} \right]$$

$$= m_0 \left[ a \left[1 - v^2/c^2\right]^{-1/2} + v \left[ -\frac{1}{\cancel{2}} \left[1 - \frac{v^2}{c^2}\right]^{-3/2} \left(-\frac{\cancel{2}v}{c^2}\right) \cdot \frac{dv}{dt}^{a} \right] \right]$$

$$= m_0 a \left[ \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \left[ 1 + \frac{v^2}{c^2} \left[1 - \frac{v^2}{c^2}\right]^{-1} \right] \right]$$

$$= \frac{m_0 a}{\sqrt{1 - v^2/c^2}} \left[ 1 + \frac{v^2}{\cancel{c^2}} \left[ \frac{\cancel{c^2}}{c^2 - v^2} \right] \right]$$

$$= \frac{m_0 a}{\sqrt{1 - v^2/c^2}} \left[ \frac{c^2 - \cancel{v^2} + \cancel{v^2}}{c^2 - v^2} \right]$$

$$= \frac{m_0 a}{\sqrt{1 - v^2/c^2}} \left[ \frac{1}{1 - v^2/c^2} \right]$$

$$\boxed{F = \frac{m_0 a}{(1 - v^2/c^2)^{3/2}}}$$

Q15. (a) The density of an object on earth is $\rho_0$. What density would be found by an observer on the earth if the object is moving at a relative speed $v$?

(b) The density of lead on the earth is $1.1\times10^3\ kg/m^3$. What density would be found if lead object is moving at 0.8c?

(a) Rest mass per unit rest frame volume $\rho_0 = m_0/V_0$

Rest mass per unit observer frame volume $D = \dfrac{m_0}{(V_0/\gamma)} = \gamma\rho_0$

Relativistic mass per unit observer frame vol. $D = \dfrac{\gamma m_0}{V_0/\gamma} = \gamma^2\rho_0$

Since observer is on earth (considered rest frame)

Rest mass per unit observer frame volume $= \boxed{\gamma\rho_0}$

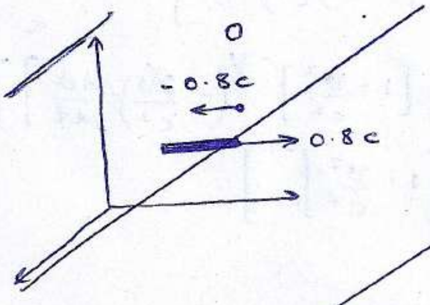
(b)
$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{0.64c^2}{c^2}}} = \frac{1}{0.6} \simeq 1.67.$$

$$\rho = \gamma\rho_0 = 1.67\times 11\times10^3\ kg/m^3$$
$$\boxed{= 18.37\times10^3\ kg/m^3}$$

Q16. A rod of length 90 cm (in its rest frame) is travelling along its length with a speed of 0.8c in frame O. A particle moving in direc$^n$ opposite to that of rod with speed 0.8c in frame O passes the rod. How much time will the particle take to cross the rod.

(a) in a frame O

(b) in the rest frame of particle.

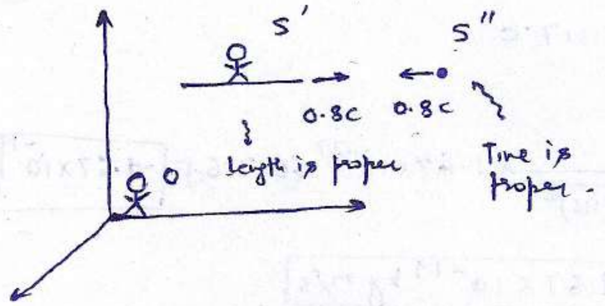



O is moving with $v = -0.8c$ wrt rod

O is moving with $v = +0.8c$ wrt particle

$\therefore$ relative velocity b/w rod & particle observed by O
$$= \frac{-(0.8c+0.8c)}{1 - \frac{(-0.8)(+0.8)\cancel{c^2}}{\cancel{c^2}}}$$
$$= \frac{-1.6c}{1+0.64} \simeq -0.976c.$$

$\therefore$ Time taken by particle to cross rod
$$t = \frac{0.9m}{0.976c\ m/s} = 3.07\ ns.$$




relative velocity b/w S' & S'' as by O

$v = 0.8\ m/s \qquad u_x = -0.8c.$

$$u_x' = \frac{-0.8c - 0.8c}{1 - \frac{(-0.8)(0.8)c^2}{c^2}}$$

$$= \frac{1.6}{1+0.64}c = 0.976c$$

To find time intervals in all three frames, we need $\gamma s$.

$$\gamma_{os'} = \frac{1}{\sqrt{1-(0.8)^2}} = \frac{5}{3}$$

$$\gamma_{os''} = \frac{1}{\sqrt{1-(0.8)^2}} = \frac{5}{3}$$

$$\gamma_{ss''} = \frac{1}{\sqrt{1-(0.976)^2}} = 4.59$$

$$\Delta t' = \frac{L'}{u_x'} = \frac{0.9m}{0.976c\ m/s} = \boxed{3.07\ ns}$$

$$\Delta t'' = \frac{\Delta t'}{\gamma_{ss''}} = \frac{3.07\ ns}{4.59} = \boxed{0.81\ ns}$$

$$\Delta t_o = \gamma_{os''} \times \Delta t'' = \frac{5}{3} \times 0.81 = \boxed{1.34\ ns}$$

Q 17. A proton (of proper mass $= 1.67 \times 10^{-27}$ kg) is moving with velocity $(u_x, u_y, u_z)$ where $u_x = c/2$, $u_y = c/3$, $u_z = -c/3$ wrt an observer O. Another observer O' is moving wrt O, with velocity of $c/\sqrt{10}$ in the x-direc$^n$. Calculate the three components of linear momentum of proton wrt O and O'.

wrt O,

$$p_x = \gamma_x m_o v_x = \frac{1}{\sqrt{1-(0.5)^2}} \times 1.67 \times 10^{-27} \times \frac{c}{2} = \boxed{\cancel{9.88}\ 2.89 \times 10^{-19}\ kg\,m/s}$$

$$p_y = \gamma_y m_o v_y = \frac{1}{\sqrt{1-(0.33)^2}} \times 1.67 \times 10^{-27} \times \frac{c}{3} = \boxed{1.76 \times 10^{-19}\ kg\ m/s}$$

$$p_z = \gamma_z m_o v_z = \frac{1}{\sqrt{1-(0.33)^2}}\ 1.67 \times 10^{-27} \times \frac{-c}{3} = \boxed{-1.76 \times 10^{-19}\ kg\,m/s}$$

wrt O'

$$v_x' = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} = \frac{0.5c - c/\sqrt{10}}{1 - \frac{0.5c^2}{\sqrt{10}c^2}} = \frac{0.184c}{0.582} = 0.316c$$

$$u_y' = \frac{u_y - v}{1 - \frac{u_y v}{c^2}} = \frac{0.33c - c/\sqrt{10}}{1 - \frac{0.33c^2}{\sqrt{10}c^2}} = \frac{0.017}{0.0537} = -0.317c$$

$$u'_z = \frac{u_z - v}{1 - \frac{u_z v}{c^2}} = +0.317c$$

$$\therefore p'_x = \gamma m_o u'_x = \frac{1}{\sqrt{1+(0.316)^2}} \times 1.67\times10^{-27} \times (0.316c) = \boxed{-1.67\times10^{-19}\ kg\,m/s}$$

$$p'_y = \gamma m_o u'_y \approx \boxed{-1.67\times10^{-19}\ kg\,m/s}$$

$$p'_z = \gamma m_o u'_z \approx \boxed{+1.67\times10^{-19}\ kg\,m/s}$$

Q18. A pion has a proper mass = 280 $m_e$, where $m_e$ = proper mass of electron. While at rest, the pion decays into two photons, one of which moves with velocity $(u_x, u_y, u_z)$ where $u_x = -c/2$, $u_y = 0$. Calculate $u_z$. Find the velocity components of other photon. Calculate wavelength of each photon.

$m_p = 280\, m_e$

$u_x = -c/2$; $u_y = 0$; $u_z = ?$

for first photon

$$u_x^2 + \cancel{u_y^2}^{\,0} + u_z^2 = c^2.$$

$$u_z^2 = c^2 - u_x^2 = c^2 - \left(-\frac{c}{2}\right)^2 = \frac{3c^2}{4}$$

$$\boxed{u_z = \frac{\sqrt{3}c}{2}}$$

By conservation of momentum, second photon equal & opposite to first photon.

$$\therefore u'_x = -u_x = -\left(-\frac{c}{2}\right) = \frac{c}{2}$$

$$u'_y = -u_y = 0$$

$$u'_z = -u_z = -\frac{\sqrt{3}}{2}c.$$

$\therefore$ for second photon, $u_x', u_y', u_z'$
$$= \boxed{\left(c/2,\ 0,\ -\sqrt{3}/2\, c\right)}$$

Since each photon has same magnitude of velocity. Both should be equally energetic → ie of same frequency.

By conservation of energy,

$$\underbrace{T.E}_{\text{associated to pion}} = \underbrace{h\nu + h\nu}_{\text{photons released}}$$

$$m_o c^2 + \underbrace{K.E}_{0 \text{ bec. pion at rest}} = h\nu + h\nu$$

$$\therefore 2h\nu = m_o c^2$$

$$\nu = \frac{m_o c^2}{2h}$$

$$= \frac{280\times9.1\times10^{-31}\times9\times10^{16}}{2\times6.63\times10^{-34}}\ hz$$

$$= 1729\times10^{19}\ hz$$

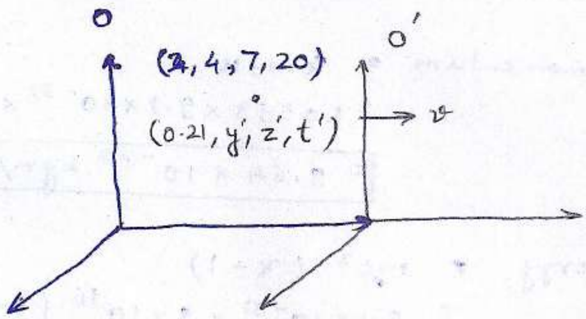
$$= 1.729\times10^{22}\ hz$$

$$c = \nu\lambda.$$

$$\lambda = c/\nu = \frac{3\times10^8\ m/s}{1.729\times10^{22}\ s^{-1}}$$

$$\boxed{\lambda = 1.735\times10^{-14}\ m}$$

Q19 An event happens at (2m, 4m, 7m, 20ns) according to an observer O. Another observer O' is moving wrt O, with uniform velocity $v$, in the common $x$-direction. The observer O' finds the value of $x' = 21$ cm. Find all the other coordinates of event according to O' using Galilean transformations and well as Lorentz transformations.




using Lorentz transform

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}\ ; \quad y' = y\ ; \quad z' = z\ ; \quad t' = \frac{t - \frac{vx}{c}}{\sqrt{1 - v^2/c^2}}$$

and invariancy of wavefront

$$x^2 + y^2 + z^2 - c^2t^2 = x'^2 + y'^2 + z'^2 + c^2t'^2$$

$$x^2 - c^2t^2 = x'^2 - c^2t'^2$$

$$x^2 - x'^2 = c^2(t^2 - t'^2)$$

$$2^2 - (0.21)^2 = c^2\left((20\times10^{-9})^2 - t'^2\right)$$

$$\frac{(2+0.21)(2-0.21)}{(3\times10^8)^2} = (20\times10^{-9})^2 - t'^2$$

$$400\times10^{-18} - \frac{(2.21)(1.79)}{9\times10^{16}} = t'^2$$

$$t'^2 = 4\times10^{-16} - 0.4395\times10^{-16}$$

$$t' = \sqrt{3.56\times10^{-16}} \approx \boxed{1.89\times10^{-8}\ \text{sec.}}$$

Lorentz $\therefore$ $\boxed{x' = 0.21\text{m};\ y' = 4\text{m};\ z' = 7\text{m};\ t' = 18.9\text{ns}}$

using Galilean transformation, $x' = x - vt$, $y' = y$, $z' = z$, $t' = t - \frac{vx}{c^2}$

$0.21 = 2 - v\times20\times10^{-9} \rightarrow v = 8.95\times10^7$ m/s $\rightarrow t' = 20\text{ns} - 1.98\text{ ns} \approx 18\text{ns}$.

Galilean: $\boxed{x' = 0.21\text{m};\ y' = 4\text{m};\ z' = 7\text{m},\ t' = 18\text{ns}}$

Q20 An electron with proper mass $= 9.1 \times 10^{-31}$ kg is moving with a velocity of $c/3$. Calculate its relativistic mass, linear momentum, kinetic energy and total energy

Relativistic mass $= \gamma m_0$  $\gamma = \frac{1}{\sqrt{1-(0.33)^2}} = 1.0593$

$= 1.0593 \times 9.1 \times 10^{-31}$ kg

$= \boxed{9.64 \times 10^{-31}\ \text{kg}}$

Relativistic momentum $= \gamma m_0 u$

$= 1.0593 \times 9.1 \times 10^{-31} \times \frac{3 \times 10^8}{3}$

$\boxed{= 9.64 \times 10^{-23}\ \text{kgm/s}}$

Kinetic energy $= m_0 c^2 (\gamma - 1)$

$= 9.1 \times 10^{-31} \times 9 \times 10^{16} (1.0593 - 1)$

$\boxed{= 4.85 \times 10^{-15}\ \text{J}} = \boxed{30.3\ \text{keV}}$

Total energy $= \gamma m c^2$

$= 1.0593 \times 9.1 \times 10^{-31} \times 9 \times 10^{16}$

$\boxed{= 86.76 \times 10^{-15}\ \text{J}} = \boxed{0.542\ \text{MeV}}$

Q 21. Show that using Lorentz transformations:

$$x^2 + y^2 + z^2 - c^2t^2 = x'^2 + y'^2 + z'^2 - c^2t'^2$$

Acc. to LT, $x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$ ; $y' = y$, $z' = z$ ; $t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - v^2/c^2}}$

$\therefore$ RHS $= x'^2 + y'^2 + z'^2 - c^2t'^2 = \left(\frac{x - vt}{\sqrt{1 - v^2/c^2}}\right)^2 + y^2 + z^2 + \left(\frac{t - \frac{vx}{c^2}}{\sqrt{1 - v^2/c^2}}\right)^2$

$$= \frac{(x - vt)^2 + \left(t - \frac{vx}{c^2}\right)^2}{1 - v^2/c^2} + y^2 + z^2$$

$= \cancel{\frac{x^2 + v^2t^2 - 2xvt + t^2 + \frac{v^2x^2}{c^4} - \frac{2tvx}{c^2}}{1 - \frac{v^2}{c^2}} + y^2 + z^2}$

$= \cancel{x^2 \left[1 + v^2/c^4\right] - 2xvt \left[1 + 1/c^2\right] + t^2\left[v^2 + 1\right]}$

$$= \frac{x^2 \cancel{(1 - v^2/c^2)} - c^2t^2 \cancel{(1 - v^2/c^2)}}{\cancel{(1 - v^2/c^2)}} + y^2 + z^2$$

$= x^2 + y^2 + z^2 - c^2t^2 = $ LHS $\therefore$ proved.

Q22. A beam of μ-meson travels with a speed (v) of 0.6c. Their mean life time as observed in the laboratory is $2.9 \times 10^{-6}$s. What is their mean life time at rest.

Time dilation

$$\Delta t = \gamma \Delta t'$$

$$\gamma = \frac{1}{\sqrt{1-(0.6)^2}} = \frac{1}{0.8} = \frac{5}{4}$$

$$\Delta t = \frac{5}{4} \times 2.9 \times 10^{-6}$$

$$= \boxed{3.625\ \mu s}$$

Q23. The rest mass of an electron is $9.1 \times 10^{-31}$ kg. What will be its mass when it is moving with speed 0.8c

relativistic mass

$$m = \gamma m_0 \qquad \gamma = \frac{1}{\sqrt{1-(0.80)^2}} = \frac{1}{0.6} = \frac{5}{3}$$

$$m = \frac{5}{3} \times 9.1 \times 10^{-31}\ kg$$

$$\boxed{= 1.52 \times 10^{-30}\ kg}$$

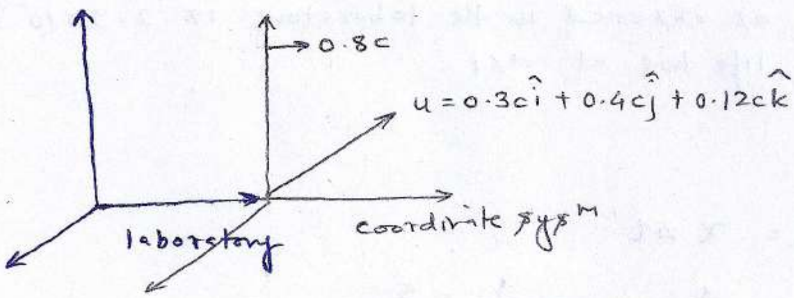
Q24. A 2 m long stick, when at rest, moves past an observer on ground with a speed 0.5c. What is the length measured by the observer?

Length contraction

$$L' = \frac{L}{\gamma} \qquad \gamma = \frac{1}{\sqrt{1-(0.5)^2}} \approx 1.155$$

$$L' = \frac{2}{1.155} \approx \boxed{1.732\ m}$$

Q25 A Particle has velocity $u = 0.3c\hat{i} + 0.4\hat{j} + 0.12c\hat{k}$ in a coordinate system moving with a velocity of 0.8 c relative to laboratory along the positive direction of x-axis. Find u in laboratory frame.




$$\gamma = \frac{1}{\sqrt{1-(0.8)^2}} = \frac{5}{3}$$

$$v = \sqrt{(0.3c)^2+(0.4c)^2+(0.12c)^2}$$
$$= \sqrt{0.09+0.16+0.0144}\,c$$
$$= 0.514c$$

(i) $$u_x = \frac{u_x' + v}{1+\frac{u_x' v}{c^2}} = \frac{0.3c + 0.8c}{1+\frac{0.51c\,0.8c}{c^2}} \approx \frac{1.1c}{1.41} \approx \boxed{0.78c}$$

(ii) $$u_y = \frac{u_y' + v}{1+\frac{u' v}{c^2}} = \frac{0.4c+0.8c}{1+\frac{0.51c \times 0.8c}{c^2}} \approx \frac{1.2c}{1.41} \approx \boxed{0.851c}$$

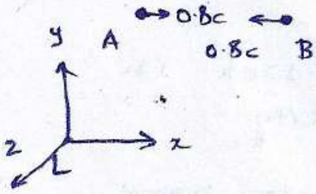
(iii) $$u_z = \frac{u_z' + v}{1+\frac{u' v}{c^2}} = \frac{0.12c+0.8c}{1+\frac{0.51c\times 0.8c}{c^2}} = \frac{0.92c}{1.41} \quad \boxed{0.65c}$$

Q26 A beam of $\mu$-meson travels with speed $v$ of 0.6c. Their mean life time observed in laboratory is $2.9 \times 10^{-6}$ s. What is their mean time at rest

Time dilation,
$$\Delta t = \gamma \Delta t' \qquad \gamma = \frac{1}{\sqrt{1-(0.6)^2}} = \frac{1}{0.8} = \frac{5}{4}$$
$$= \frac{5}{4} \times 2.9 \times 10^{-6}\,s$$
$$= \boxed{3.625\ \mu s}$$

Q27. Two particles A & B are moving in opposite direc^n each with speed of 0.8c, in laboratory frame of reference. Find the velocity of one particle relative to other.

Applying Lorentz velocity transform,




$$\underbrace{v}_{\text{vel. of A wrt L}} = +0.8c, \quad \underbrace{u_x}_{\text{vel. of B wrt L}} = -0.8c$$

$$\underbrace{u_x'}_{\text{vel. of B wrt A}} = \frac{u_x - v}{1-\frac{u_x v}{c^2}} = \frac{-0.8c-0.8c}{1-\frac{(-0.8\cancel{c})(0.8\cancel{c})}{\cancel{c^2}}}$$

$$= \frac{-1.6c}{1+0.64} = \boxed{-0.9756c}$$

Q28. Derive the mass-energy relation or prove the ~~equation~~ relation

$$E^2 - c^2 p^2 = m_0^2 c^4$$

By mass-energy equivalence,

$$E = mc^2 = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}} \quad ; \quad p = mv = \frac{m_0 v}{\sqrt{1 - v^2/c^2}}$$

kinetic energy ; relativistic mom.

$$E^2 = \frac{m_0^2 c^4}{1 - v^2/c^2} \quad ; \quad p^2 = \frac{m_0^2 v^2}{1 - v^2/c^2}$$

Multiply $p^2$ by $c^2$

$$p^2 c^2 = \frac{m_0^2 v^2 c^2}{1 - v^2/c^2}$$

do $(E)^2 - p^2c^2$

$$\therefore E^2 - p^2c^2 = \frac{m_0^2 c^4 - m_0^2 v^2 c^2}{1 - v^2/c^2}$$

$$= \frac{m_0^2 c^2 (c^2 - v^2)}{c^2 - v^2/c^4} = m_0^2 c^4$$

$$\boxed{E^2 - p^2c^2 = m_0^2 c^4} \quad \therefore \text{proved}$$

Q29. Coordinates of an event are (4m, 5m, 1m, 12ns). In another frame the coordinates are (3m, y, 1m, 8ns). Calculate y, using relativity.

By LT, $x' = \gamma(x - vt)$ ; $t' = \gamma\left(t - \frac{vx}{c^2}\right)$

$$\frac{x'}{t'} = \frac{x - vt}{t - \frac{vx}{c^2}}$$

$$\frac{3}{8 \times 10^{-9}} = \frac{4 - v \times 12 \times 10^{-9}}{12 \times 10^{-9} - \frac{v \times 4}{9 \times 10^{16}}} \rightarrow \frac{3}{8} \times 10^9 \left[12 \times 10^{-9} - \frac{4v}{9 \times 10^{16}}\right] = 4 - 12v \times 10^{-9}$$

$$\frac{36}{8} - \frac{3 \times 4 \times 10^9}{8 \times 9 \times 10^{16}} v = 4 - 12v \times 10^{-9}$$

$$v\left[12 \times 10^{-9} - \frac{1}{6 \times 10^7}\right] = 4 - \frac{36}{8} \rightarrow \frac{v}{6 \times 10^7}\left[12 \times 6 \times 10^{-2} - 1\right] = \frac{32 - 36}{8}$$

$$\therefore v = \frac{-1 \times 6 \times 10^7}{-1 + 0.72} = 1.07 \times 10^8 \text{ m/s}$$

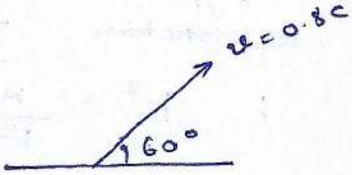
By L.T,

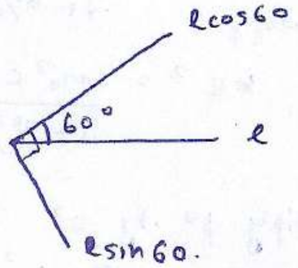
$$y' = \gamma(y - vt)$$

$$y' = \frac{1}{\sqrt{1 - \left(\frac{1.07}{3}\right)^2}}\left(5 - 1.07 \times 10^8 \times 12 \times 10^{-9}\right) = 1.07\left[5 - 1.284\right]$$

$$\boxed{\therefore y' = 3.976\,m}$$

Q30. Calculate the percentage contraction of a rod moving with a velocity 0-8 times the velocity of light in a direcn at 60° to its own length.




$\gamma = \frac{1}{\sqrt{1+(0.8)^2}} = \frac{1}{0.6}$

component of length along velocity = $l\cos 60$
component of length ~~along~~ perpendicular to velocity = $l\sin 60$
contracted length along velocity $l_x' = \frac{l\cos 60}{\cancel{\gamma}\ 1/0.6}$

$= 0.6l/2 = 0.3l$.

new length $= \sqrt{l_x'^2 + l_y^2}$

$= \sqrt{(0.3l)^2 + (l\sin 60)^2}$

$= \sqrt{0.09l^2 + 3l/4\, l^2}$.

$\simeq 0.917l$

$\therefore$ % contraction $= \frac{l - 0.917l}{l} \times 100 = \boxed{8.3\%}$

——— X ———

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