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ASSIGNMENT - 3

ENGINEERING PHYSICS

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SECTION C-72.
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Q1 Define fermi energy at zero kelvin, show that fermi energy of free electron at absolute zero is

$$E_F = \frac{h^2}{2m}\left(\frac{3n}{8\pi}\right) \quad \text{where } n = \frac{N}{V}$$

Fermi energy refers to the energy difference between the highest and lowest occupied single particle states in a quantum system of non interacting fermions at absolute zero temperatures. It is the fermi level (derived from fermi dirac statistics) at 0K or the maximum kinetic energy of an electron at 0k.

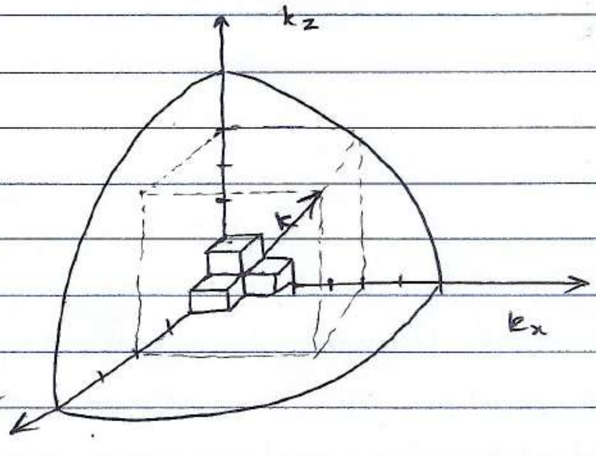
The solid can be modelled as an infinite quantum well in which electrons with effective mass m, are free to move. The energy of well is set to zero. The ~~solid~~ crystal is assumed a cube of length L. This assumption DOES NOT affect the result since the density of states per unit volume should ~~be~~ not depend on actual size or shape of of semiconductor. The solution of schrödinger wave eq^n when $V(x) = 0$ is

$$\psi = A\sin(k_x x) + B\cos(k_x x)$$

where A & B are to be determined. The wavefunction must be zero at ∞ barriers of the well. At $x = 0$, the wavefunc^n must be 0 so that only sine func^n can be valid sol^n or $B = 0$. At $x = L$, the wave func^n must also be 0 yielding following possible values for wave number k_x

$$k_x = \frac{n\pi}{L} \qquad n = 1, 2, 3, \ldots$$

This analyses can be repeated in y and z direc^{n}s. Each possible sol^n then corresponds to a cube in k-space with size $n\pi/L$



The total no. of sol^n with different value for k_x, k_y, k_z and with a magnitude of the wave vector less than k is obtained by calculating the volume of positive quadrant of sphere with radius k and dividing it by volume of single sol^n $\left(\frac{\pi}{L}\right)^3$

To. No. of states, $N = \underbrace{2}_{\text{To account two possible spins of electron in each state}} \times \underbrace{\frac{1}{8}}_{\text{only the quadrant}} \times \underbrace{\left(\frac{L}{\pi}\right)^3}_{1/\text{vol. of sol}^n} \times \underbrace{\frac{4}{3}\pi k^3}_{\text{volume of sphere}}$

Density per unit energy

$$\frac{dN}{dE} = \frac{dN}{dk} \times \frac{dk}{dE} = \left(\frac{L}{\pi}\right)^3 \pi k^2 \frac{dk}{dE}.$$

The kinetic energy E of a particle with mass m is related to wave number k, by.

$$E(k) = \frac{\hbar^2 k^2}{2m}, \text{ providing } \frac{dk}{dE} = \frac{m}{\hbar^2 k} \ \& \ k = \frac{\sqrt{2mE}}{\hbar}$$

Density of states per unit volume per unit energy,

$$g(E) = \frac{1}{L^3}\frac{dN}{dE} = \frac{1}{L^3} \times \left(\frac{L}{\pi}\right)^3 \pi k^2 \times \frac{m}{h^2 k}$$

$$= \frac{1}{\pi^2} \frac{m k^{\cancel{2}}}{h^2 \cancel{k}} = \frac{m}{\pi^2 h^2} \frac{\sqrt{2mE}}{\cancel{\hbar} \to h/2\pi}$$

$$g(E) = \frac{8\pi\sqrt{2}}{h^3} m^{3/2} E^{1/2}$$

Substituting density of states in fermi dirac distrib^n fuc^n.

$$N(E)dE = F(E)g(E)dE$$

$$= \frac{4\pi V}{h^3}(2m)^{3/2} E^{1/2} \frac{dE}{\exp\left(\frac{E-E_F}{kT}\right)+1}$$

Since at absolute zero $E = E_F \rightsquigarrow F(E) = 1$

$$\int N(E)dE = \frac{4\pi V}{h}(2m)^{3/2} \int_0^{E_F} E^{1/2} dE.$$

$$N = \frac{2}{3} \times \frac{4\pi V}{h^3} (2m)^{3/2} E_F^{3/2}$$

No. of electrons n / unit volume = N/V.

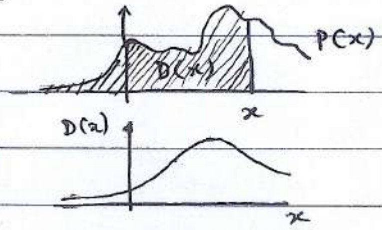
$$\therefore \boxed{E_F = \frac{h^2}{2m}\left(\frac{3n}{8\pi}\right)^{2/3} J}$$

Q2 What is the meaning of distribution function? Write three distribution functions. Specify the particle to which they apply.

The distribution function $D(x)$ describes the probability that a variate X takes on a value less than or equal to a number x.

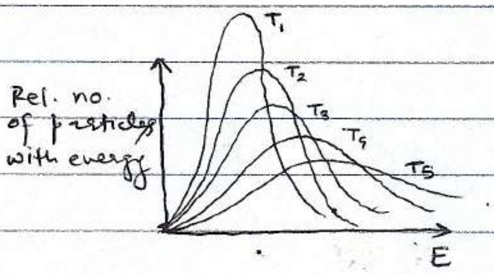
Distribution function is therefore related to a continuous probability density function $P(x)$ by

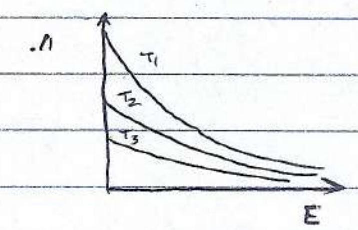
$$D(x) = P(X \le x)$$
$$= \int_{-\infty}^{x} P(u)\,du$$

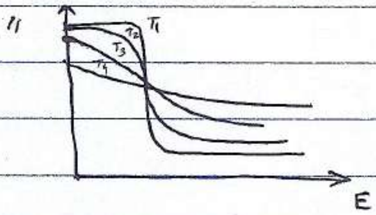


Distribution functions in statistical mechanics

MAXWELL BOLTZMANN	BOSE EINSTEIN	FERMI DIRAC
$f(\varepsilon) = Ae^{-\varepsilon/kT}$	$f(\varepsilon) = \dfrac{1}{e^{\alpha}e^{\varepsilon/kT} - 1}$	$f(\varepsilon) = \dfrac{1}{\exp\left[\dfrac{\varepsilon - \varepsilon_F}{k_B T}\right] + 1}$







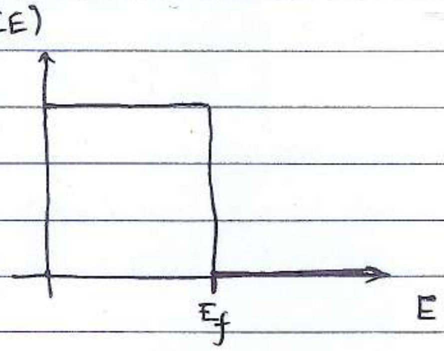
(Distinguishable)	(indistinguishable)	(indistinguishable)
Gas molecules at low densities	Bosons (photons, quarks, taus) etc.	fermions (electrons - Leptons etc)
spin does not matter	integer spin 0, 1, 2...	half integer spin ½, 3/2, 5/2.
localised particles ψ do not overlap	wave functions overlap total ψ symmetric	wave functions overlap total ψ asymmetric

Q3. Plot for low temperature, the fermi-dirac distribution function, with energy. As temperature, how is the graph modified.

$$f(E) = \frac{1}{e^{(E-E_F)/kT} + 1} \quad \text{(Fermi-Dirac prob. func}^n\text{)}$$

for $T = 0\,K$

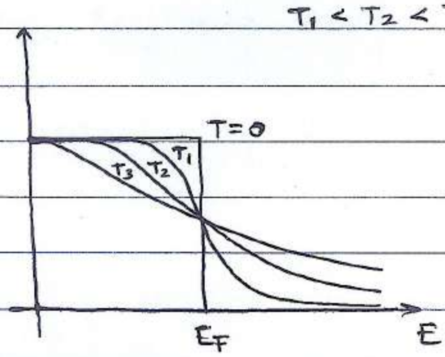
$$f(E) = \begin{cases} 0 & \text{if } E > E_F \\ 1 & \text{if } E < E_f \end{cases}$$



for $T > 0\,K$ (low temp $T < 10^4\,k$)

$$\begin{cases} 0.5 > f(E) > 0 & \text{if } E > E_f \\ f(E) = 0.5 & \text{if } E = E_f. \\ 0.5 < f(E) < 1 & \text{if } E < E_f. \end{cases}$$

$T_1 < T_2 < T_3$



Q4 Do electrons have zero energy at 0K? If not explain why?

There are vibrations in a particle, some are caused by thermal excitation while some are INTRINSICALLY QUANTUM MECHANICAL which do not stop at 0K. According to Heisenberg's uncertainty principle confining electron to a specific position (ie at rest) implies large uncertainty in its momentum and energy, thus it cannot have fixed zero energy even at 0 K.

Q5. Assuming that in tungsten (Atomic weight 183.8 g/mol, density = 19.3 g/cm³), there are two electrons per atom. Calculate the free electron density in the metal and fermi energy.

No. of atoms / unit volume $= \dfrac{19.3 \text{ g/cm}^3 \times N_A \text{ atoms/mol}}{183.8 \text{ g/mol}}$

$\simeq 0.105 \times 6.022 \times 10^{23}$ atoms/cm³

$\approx 6.32 \times 10^{22}$ atoms/cm³

free electrons / atom = 2

No. of electrons / unit volume $= 2 \times 6.32 \times 10^{22}\ e^-/\text{cm}^3$

$= 1.264 \times 10^{23}\ e^-/\text{cm}^3$

$n = \boxed{1.264 \times 10^{29}\ e^-/\text{m}^3}$

Fermi energy is given by $E_F = \dfrac{h^2}{2m}\left(\dfrac{3n}{8\pi}\right)^{2/3}$

Which for electron, reduces to $E_f = 3.65 \times 10^{-19}\ n^{2/3}$ eV

$= 3.65 \times 10^{-19} \times (1.264 \times 10^{29})^{2/3}$ eV

$= 3.65 \times 10^{-19} \times 2.51 \times 10^{19}$ eV

$= \cancel{9.16 \text{ eV}}$ 4.58 eV ?

Q6 Differentiate between MB, BE & FD statistics

	Maxwell Boltzmann	Bose-Einstein	Fermi-Dirac
Applies to systems of	Identical, distinguishable particles	Identical, Non distinguishable do not obey excl. principle	Identical, Non distinguishable that obey excl. principle.
Category of particles	classical	Bosons	fermions
properties of particles	Any spin Particles far apart so that wave func's do not overlap.	spin 0, 1, 2... wave func's are symmetric to interchange of particle labels	spin 1/2, 3/2, 5/2 wave func's are asymmetric to interchange of particle labels

Examples	Molecules of a gas	Photons in cavity; phonons in solid; liquid Helium;	Free electrons in a metal; electrons in star whose atoms are collapsed (white) (dwarf stars)
Distribution function (No. of e^- in each state of energy at temperature T)	$f_{MB}(E) = A e^{-\varepsilon/kT}$	$f_{BE}(\varepsilon) = \frac{1}{e^{\alpha} e^{\varepsilon/kT} - 1}$	$f_{FD}(\varepsilon) = \frac{1}{e^{(E-E_F)/kT} + 1}$
Properties of Distribution	No limit to no. of particles in a state	No limit to no. of particles per state. More particles/state than f_{MB} at low energies. Approaches f_{MB} at high energies.	Never more than 1 particle/state. fewer particles than f_{MB} at low energies. Approaches f_{MB} at high energies

Q7. At what temperature can we expect a 10% probability that electrons in silver have an energy which is 1% above the fermi energy? The fermi energy of silver is 5.5 eV.

$$P = \frac{1}{e^{(E-E_F)/kT} + 1}$$

$$k = 8.62 \times 10^{-5} \text{ eV/K}$$

$$E = E_F + 0.01 E_F$$

$$P = \frac{1}{10} = \frac{1}{e^{0.01 E_F/kT} + 1} \rightsquigarrow 10 - 1 = e^{1.01 E_F/kT}$$

$$\frac{0.01 E_F}{kT} = \ln 9$$

$$T = \frac{0.01 E_F}{\ln 9 \; k} = \frac{0.01 \times 5.5 \cancel{eV}}{\ln 9 \times 8.62 \times 10^{-5} \cancel{eV}/k}$$

$$\approx \boxed{290.39 \text{ k}}$$

Q8. Calculate the conductivity of pure Si at room temperature when concentration of carriers is $1.6 \times 10^{10} / cm^3$. Take $\mu_p = 1500\ cm^3/volts$ and $\mu_n = 500\ cm^2/Vs$ at room temperature.

Drift velocity of carriers in semiconductor.

$$|v| = \frac{q\tau}{m} \varepsilon = \mu \varepsilon$$

Mobility

The electric current density

$$|J| = qnv_n + qpv_p = q(n\mu_n + p\mu_p)\varepsilon.$$

Conductivity of a semiconductor is.

$$\sigma = \frac{|J|}{\varepsilon} = q(n\mu_n + p\mu_p).$$

for intrinsic semiconductor $n = p = n_i$

$$\sigma = qn_i(\mu_n + \mu_p)$$

$$= 1.6 \times 10^{-19}\ C \times 1.6 \times 10^{10}/cm^3\ (1500 + 500)\ cm^2/Vs$$

$$= 2.56 \times 10^{-9} \frac{C}{cm\ Vs} \cdot 2000$$

$$= 5.12 \times 10^{-6} \frac{A}{cm\ V}$$

$$= \boxed{5.12 \times 10^{-6}\ S/cm}$$

Q9. An electric field of 200 Vm is applied to a sample of silicon semiconductor having resistivity of $1.54 \times 10^{-8}\ \Omega m$ at a temperature 300K. Calculate the drift velocity, mobility, relaxation time of electrons assuming that there are $5.8 \times 10^{28}\ e/m^3$ and the thermal velocity of conduction e's.

given, $E = 200\ V/m$, $\rho = 1.54 \times 10^{-8}\ \Omega m$, $T = 300k$,
$n = 5.8 \times 10^{28}\ e/m^3$

$$v_D = j/ne \quad ; \quad j = \rho E = E/\rho$$

$$\therefore v_D = \frac{E}{\rho n e}$$

$$= \frac{200\ V/m}{1.54 \times 10^{-8}\ \Omega m \times 5.8 \times 10^{28}/m^3 \times 1.6 \times 10^{-19}\ C}$$

$$= \frac{200\ V/\cancel{m}}{1.54 \times 5.8 \times 1.6 \times 10^{-8+28-19}\ \cancel{\Omega}\cancel{m}\,C/\cancel{m^3}}$$ V/A

$$= \frac{200\ \cancel{V}}{14.29 \times 10^{1}\ \frac{\cancel{V}C}{Am}}$$

$$= \frac{200}{142.9}\ \frac{m\cancel{A}}{C}\ C/s$$

$$\simeq \boxed{1.4\ m/s}$$

$$\mu = \frac{V_d}{E} = \frac{1.4\ m/s}{200\ V/m} = \boxed{0.007\ m^2/Vs}$$

$$\mu = \frac{e\tau}{m} \rightsquigarrow \tau = \frac{m\mu}{e} = \frac{9.1 \times 10^{-31}\ kg \times 0.007\ m^2/Vs}{1.6 \times 10^{-19}\ C}$$

$$\approx 0.04 \times 10^{-31+19}\ \frac{kg\,m^2}{\cancel{C} \times \frac{J}{\cancel{C}} s}$$

$$= 0.04 \times 10^{-12}\ \frac{\cancel{kg}\cancel{m^2}}{\cancel{kg}\cancel{m^2}/s^2 \times s}$$

$$= \boxed{4 \times 10^{-14}\ s}$$

Thermal velocity $v_t = \sqrt{\frac{3k_BT}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23}) \times 300}{9.1 \times 10^{-31}}}$

$$\approx \sqrt{136.5 \times 10^{8}} \approx \boxed{1.17 \times 10^{5}\ m/s}$$

Q10. Show that the average kinetic energy ~~is~~ in a fermi gas is $\frac{3}{5} E_f$

Average KE of e^- is given by.

$$\bar{E} = \frac{1}{N} \int_0^{E_f} E\, N(E)\, dE$$

$$= \frac{4\pi V}{N h^3} (2m)^{3/2} \int_0^{E_f} E^{3/2} dE$$

$$= \frac{2}{5} \frac{4\pi V}{N h^3} (2m)^{3/2} E_F^{5/2}.$$

$$\left(N = \frac{2}{3} \times \frac{4\pi V}{h^3} (2m)^{3/2} E_F^{3/2} \right)$$

$$= \frac{\cancel{2}}{5} \times \frac{\cancel{4\pi V} \cancel{(2m)^{3/2}} E^{\cancel{5/2}}}{\frac{\cancel{2} \times \cancel{4\pi V} \cancel{(2m)^{3/2}} \cancel{E_F^{3/2}} \cancel{h^3}}{3 \cancel{h^3}}}$$

$$\boxed{\bar{E} = \frac{3}{5} E_F}$$

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