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ASSIGNMENT - 1

ENGINEERING MATHEMATICS

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SECTION C-72
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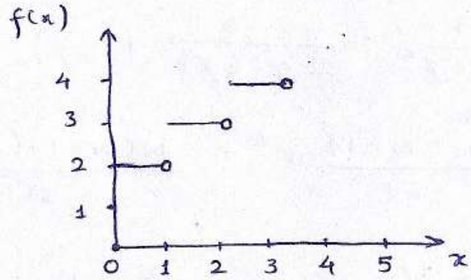
Q1. Find the laplace transform of $f(x) = [x+2]$, where [ ] stands for the greatest integer function.



$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt$$

$$= \int_0^1 e^{-st}.2dt + \int_1^2 e^{-st} 3\,dt + \dots$$

$$= 2\left[\frac{e^{-st}}{-s}\right]_0^1 + 3\left[\frac{e^{-st}}{-s}\right]_1^2 + \dots$$

$$= \frac{2e^{-s}}{-s} - \frac{2}{-s} + \frac{3e^{-2s}}{-s} - \frac{3e^{-s}}{-s} + \frac{4e^{-3s}}{-s} - \frac{4e^{-2s}}{-s} + \dots$$

$$= \frac{-2}{-s} - \frac{1}{s}\left[2e^{-s} + 3e^{-2s} - 3e^{-s} + 4e^{-3s} - 4e^{-2s} + \dots\right]$$

$$= \frac{2}{s} - \frac{1}{s}\left[-e^{-s} - e^{-2s} - e^{-3s} - \dots\right]$$

$$= \frac{2}{s} + \frac{1}{s}\left[e^{-s} + e^{-2s} + e^{-3s} + \dots\right]$$

$$= \frac{2}{s} + \frac{1}{s}\left[\frac{e^{-s}}{1-e^{-s}}\right]$$

$$= \frac{1}{s}\left[\frac{2-2e^{-s}+e^{-s}}{1-e^{-s}}\right]$$

$$= \boxed{\frac{1}{s}\,\frac{2-e^{-s}}{1-e^{-s}}}$$

Q2 Find the laplace transform of $e^{-mx} J_0(nx)$, where

$$J_0(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(k!)^2}\left(\frac{x}{2}\right)^{2k}$$

$$\mathcal{L}\{J_0(x)\} = \int_0^{\infty} e^{-st} J_0(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(k!)^2 \cdot 2^{2k}} \int_0^{\infty} x^{2k} e^{-st}\,dt$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{(k!)^2 2^{2k}} \cdot \frac{(2k)!}{s^{2k+1}} \qquad \left[\text{ie } \mathcal{L}\{t^n\} = \frac{\Gamma n+1}{s^{n+1}}\right]$$

$$= \frac{1}{s} \sum_{k=0}^{\infty} \frac{(-1)^k}{(k!)^2 2^{2k}} \cdot \frac{(2k)!}{s^{2k}}$$

$$= \frac{1}{s}\left[\frac{-1^0}{(0!)^2 2^0} \cdot \frac{0!}{s^0} + \frac{(-1)^1}{1!\,2^2}\frac{2!}{s^2} + \frac{(-1)^2}{(2!)^2 2^4} \cdot \frac{4!}{s^4} + \frac{(-1)^3}{(3!)^3 \cdot 2^6} \cdot \frac{6!}{s^6} + \cdots\right]$$

$$\Rightarrow \cancel{\frac{1}{s}\left[1 - \frac{1}{2s^2} + \frac{3}{8s^4} - \frac{5}{16s^6} + \cdots\right]}$$

$$= \frac{1}{s}\left[1 - \frac{1}{2s^2} + \frac{1 \cdot 3}{2 \cdot 4 s^4} - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 s^6} + \cdots\right]$$

$$(1+x)^N = 1 + Nx + \frac{N(N-1)}{2!}x^2 + \frac{N(N-1)(N-2)}{3!}x^3 + \cdots$$

$$Nx = -\frac{1}{2s^2} \rightsquigarrow N^2 x^2 = \frac{1}{4s^4}$$

$$\frac{3}{8s^4} = \frac{N(N-1)}{2!}x^2$$

$$\frac{\cancel{N}(N-1)\cancel{x^2}/2!}{N^{\cancel{2}}\cancel{x^2}} = \frac{3/8s^4}{1/4s^4}$$

$$\frac{N-1}{2N} = \frac{3}{2} \rightsquigarrow 2N - 2 = 6N$$

$$-4N = 2$$

$$N = \underline{\underline{-1/2}} \qquad -\tfrac{1}{2}x = -\frac{1}{2s^2}$$

$$= \frac{1}{s}\left(1 + \frac{1}{s^2}\right)^{-1/2}$$

change of scale $\rightsquigarrow \mathcal{L}\{f(at)\} = \frac{1}{a}F(s/a)$

$$\mathcal{L}\{J_0(nx)\} = \frac{1}{\cancel{n}} \frac{1}{s/\cancel{n}}\left[1 + \frac{1}{(s/n)^2}\right]^{-1/2}$$

first shif $\to \mathcal{L}\{e^{+at} f(t)\} = F(s-a)$

$$\boxed{\mathcal{L}\{e^{-mx} J_0(nx)\} = \frac{1}{s+m}\left[1 + \frac{n^2}{(s+m)^2}\right]^{-1/2}}$$

Q3. Find inverse laplace transform of

$$\frac{s}{s^4 + s^2 + 1}$$

$$\frac{s}{s^4+s^2+1} = \frac{s}{s^4+2s^2+1-s^2} = \frac{s}{(s^2+1)^2 - s^2}$$

$$= \frac{s}{(s^2+1+s)(s^2+1-s)}$$

$$= \frac{As+B}{s^2+1+s} + \frac{Cx+D}{s^2+1-s}$$

$$= \frac{(As+B)(s^2+1-s) + (Cx+D)(s^2+1+s)}{(s^2+1+s)(s^2+1-s)}$$

$$= \frac{As^3 + As - As^2 + Bs^2 + B - Bs + Cs^3 + Cs + Cs^2 + Ds^2 + D + Ds}{(s^2+1+s)(s^2+1-s)}$$

$$= \frac{s^3(A+C) + s^2(-A+B+C+D) + s(A-B+C+D) + B + D}{(s^2+1+s)(s^2+1-s)}$$

comparing,

$$A + C = 0 \qquad\qquad \rightsquigarrow A = 0$$
$$-A + B + C + D = 0 \qquad\qquad B = -1/2$$
$$A - B + C + D = 1 \qquad\qquad C = 0$$
$$B + D = 0 \qquad\qquad D = 1/2$$

$$= \frac{-1}{2(s^2+1+s)} + \frac{1}{2(s^2+1-s)}$$

$$F(s) = -\frac{1}{2} \times \frac{1}{\left(s+\frac{1}{2}\right)^2 + \frac{3}{4}} + \frac{1}{2} \times \frac{1}{\left(s-\frac{1}{2}\right)^2 + \frac{3}{4}}$$

$\cancel{= -\frac{1}{2} e^{-1/2}}$

$$\mathcal{L}^{-1}\{F(s)\} = -\frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{(s+1/2)^2+3/4}\right\} + \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{(s-1/2)^2+3/4}\right\}$$

$$= -\frac{1}{\cancel{2}} e^{-1/2 t} \cdot \frac{\cancel{2}}{\sqrt{3}} \sin\frac{\sqrt{3}}{2}t + \frac{1}{\cancel{2}} e^{1/2 t} \frac{\cancel{2}}{\sqrt{3}} \sin\frac{\sqrt{3}}{2}t$$

$$\boxed{\mathcal{L}^{-1}\{F(s)\} = \frac{1}{\sqrt{3}} \sin\frac{\sqrt{3}}{2}t \left[e^{1/2 t} - e^{-1/2 t}\right] = f(t)}$$

Q4. Solve the following integral - differential equation using laplace transform method:

$$y(t) = e^{-t} + \int_0^t \sin(t-u)\, y(u)\, du$$

Let $\mathcal{L}\{y(t)\} = Y(s) \rightsquigarrow \mathcal{L}^{-1}\{Y(s)\} = y(t)$

Taking laplace tranformation on both sides

$$\mathcal{L}\{y(t)\} = \mathcal{L}\{e^{-t}\} + \mathcal{L}\left\{\int_0^t \sin(t-u)\, y(u)\, du\right\}$$

$$\left[\begin{array}{l} - \mathcal{L}\{e^{at}\} = 1/s-a. \\ - \mathcal{L}\left\{\int_0^t f(u)\, g(t-u)\, du\right\} = F(s)\, G(s) \quad \text{(convolation)} \\ \quad \text{where } F(s) = \mathcal{L}\{f(u)\}; \\ \quad\quad\quad\;\; G(s) = \mathcal{L}\{g(u)\} \end{array}\right]$$

$$Y = \frac{1}{s+1} + Y \times \frac{1}{s^2+1}$$

$$Y\left[1 - \frac{1}{s^2+1}\right] = \frac{1}{s+1}$$

$$Y\left[\frac{s^2+\cancel{1}-\cancel{1}}{s^2+1}\right] = \frac{1}{s+1}$$

$$Y = \frac{s^2+1}{s^2(s+1)}$$

$$Y = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1}$$

$$= \frac{A(s+1)\cdot s + B(s+1) + Cs^2}{s^2(s+1)}$$

$$= \frac{As^2 + As + Bs + B + Cs^2}{s^2(s+1)}$$

$$= \frac{s^2(A+C) + s(A+B) + B}{s^2(s+1)}$$

comparing

$A + C = 1 \rightsquigarrow A = -1$

$A + B = 0 \qquad B = 1$

$B = 1 \qquad C = 2$

$$\therefore \quad Y = \frac{-1}{s} + \frac{1}{s^2} + \frac{2}{s+1}$$

Taking $\mathcal{L}^{-1}$ on both sides

$$\mathcal{L}^{-1}\{y\} = -\mathcal{L}^{-1}\{1/s\} + \mathcal{L}^{-1}\{1/s^2\} + \mathcal{L}^{-1}\{2/s+1\}$$

soln $\boxed{y(t) = -1 + t + 2e^{-t}}$

$$\left[\begin{array}{l} \mathcal{L}\{1\} = 1/s \rightsquigarrow \mathcal{L}^{-1}\{1/s\} = 1 \\ \mathcal{L}\{t^n\} = \overline{|n+1}/s^{n+1} \rightarrow \mathcal{L}\{t\} = \frac{1!}{s^2} \rightsquigarrow \mathcal{L}^{-1}\{1/s^2\} = t \\ \mathcal{L}\{e^{at}\} = 1/s-a \rightarrow \mathcal{L}^{-1}\{1/s-a\} = e^{-at} \end{array}\right]$$

Q5. Determine the harmonic conjugate of

$$u(x, y) = x\sin x\cosh y - y\cos x\sinh y$$

using cauchy - riemann equations.

Acc. to CR equations for $f(x,y)$ to be analytic,

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad ; \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

$$\frac{\partial v}{\partial y} = \frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(x\sin x\cosh y - y\cos x\sinh y)$$

$$= (x\cos x + \sin x)\cosh y + y\sin x\sinh y$$

$$= x\cos x\cosh y + \sin x\cosh y + y\sin x\sinh y$$

integrating v wrt y.

$$v = \int x\cos x\cosh y + \sin x\cosh y + y\sin x\sinh y\, dy$$

$$= x\cos x\int\cosh y + \sin x\int\cosh y$$
$$+ \sin x\int y\sinh y + c$$

$$= x\cos x\sinh y + \sin x\sinh y$$
$$+ \sin x\left[y\int\sinh y - \int\frac{d}{dy}y\int\sinh y\right] + c$$

$$= (x\cos x + \sin x)\sinh y + \sin x\left[y\cosh y - \int\int\sinh y\right] + c$$

$$= (x\cos x + \cancel{\sin x})\cancel{\sinh y} + \cancel{\sin x}[y\cosh y - \cancel{\sinh y}] + c$$

$$\boxed{v = x\cos x\sinh y + y\sin x\cosh y + c}$$

Q6. Find the analytic function $f(z) = u + iv$ if

$$u - v = \frac{\cos x + \sin x - e^{-y}}{2\cos x - e^{y} - e^{-y}}$$

when $f(\pi/2) = 0$

Let $f(z) = u + iv$

$\therefore \quad if(z) = iu - v$

$\therefore \quad (1+i)f(z) = (u-v) + i(u+v) = U + iV$

$$\therefore \quad U = u - v = \frac{\cos x + \sin x - e^{-y}}{2\cos x - e^{y} - e^{-y}}$$

$$\frac{\partial U}{\partial x} = \frac{(2\cos x - e^{y} - e^{-y})(-\sin x + \cos x) - (\cos x + \sin x - e^{-y})(-2\sin x)}{(2\cos x - e^{y} - e^{-y})^2}$$

$$\frac{\partial U}{\partial y} = \frac{(2\cos x - e^{y} - e^{-y})(e^{-y}) - (\cos x + \sin x - e^{-y})(-e^{y} + e^{-y})}{(2\cos x - e^{y} - e^{-y})^2}$$

$$\therefore (1+i)f'(z) = \frac{\partial U}{\partial x} - \frac{i\,\partial V}{\partial x} = \frac{\partial U}{\partial x} - i\frac{\partial U}{\partial y}$$

(by CR eq^ns $\frac{\partial U}{\partial x} = \frac{\partial V}{\partial y}$; $\frac{\partial U}{\partial y} = -\frac{\partial V}{\partial x}$)

By Milne - thompson's method

put $x = z$ & $y = 0$ in $f'(z)$

$$(1+i)f'(z) = \left[\frac{(2\cos z - 2)(\cos z - \sin z) + (\cos z + \sin z - 1)(+2\sin z)}{(2\cos z - 2)^2}\right]$$

$$- i\left[\frac{\cancel{(2\cos z - 2)}}{(2\cos z - 2)^{\cancel{2}}}\right]$$

$$(1+i)f'(z) = \left[\frac{2\cos^2 z - 2\cos z - \cancel{2\cos z\sin z} + \cancel{2\sin z} + \cancel{2\sin z}\cos z + 2\sin^2 z - \cancel{2\sin z}}{(2\cos z - 2)^2}\right]$$

$$- i\left[\frac{1}{2\cos z - 2}\right]$$

$$(1+i)\, f'(z) = \frac{2-2\cos z}{(2\cos z-2)^2} - i\,\frac{1}{(2\cos z-2)}$$

$$\cancel{2-2\cos z - i(2\cos z-2)}$$

$$= \frac{-\cancel{(2\cos z-2)}}{(2\cos z-2)^{\cancel{2}}} - \frac{i}{2\cos z-2}$$

$$= \frac{1}{2\cos z-2}\,(-1-i)$$

$$= \frac{1}{2(\cos z-1)}\,(-1-i)$$

$$\cancel{(1+i)}\, f'(z) = \frac{1}{2(1-\cos z)}\,\cancel{(1+i)}$$

$$f'(z) = \frac{1}{2\times 2\sin^2 z/2}$$

$$f'(z) = \frac{1}{4}\operatorname{cosec}^2 z/2.$$

Integrating both sides wrt z

$$f(z) = \frac{1}{4}\int \operatorname{cosec}^2 \frac{z}{2}$$

$$= \frac{1}{\cancel{4}_2}\;\frac{-\cot\frac{z}{2}}{1/\cancel{2}} + c$$

$$f(z) = -\frac{1}{2}\cot\frac{z}{2} + c$$

$$f(\pi/2) = -\frac{1}{2}\cot \pi/4 + c$$

$$0 = -\frac{1}{2} + c \;\rightarrow\; c = 1/2.$$

$$\therefore \boxed{f(z) = -\frac{1}{2}\cot\frac{z}{2} + \frac{1}{2}}$$

Q7. If $f(z)$ is a regular function of z, prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4|f'(z)|^2$$

where $z = x + iy$.
$u = g(x, y)$
$v = h(x, y)$

$$f(z) = u + iv$$
$$|f(z)| = \sqrt{u^2+v^2}$$
$$|f(z)|^2 = u^2+v^2$$

LHS : $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)(u^2+v^2)$

$$= \frac{\partial}{\partial x}\frac{\partial}{\partial x}u^2 + \frac{\partial}{\partial x}\frac{\partial}{\partial x}v^2 + \frac{\partial}{\partial y}\frac{\partial}{\partial y}u^2 + \frac{\partial}{\partial y}\frac{\partial}{\partial y}v^2$$

$$= \frac{\partial}{\partial x} 2u\frac{\partial u}{\partial x} + \frac{\partial}{\partial x} 2v\frac{\partial v}{\partial x} + \frac{\partial}{\partial y} 2u\frac{\partial u}{\partial y} + \frac{\partial}{\partial y} 2v\frac{\partial v}{\partial y}$$

$$= \left(2\left(\frac{\partial u}{\partial x}\right)^2 + 2u\frac{\partial^2 u}{\partial x^2}\right) + \left(2\left(\frac{\partial v}{\partial x}\right)^2 + 2v\frac{\partial^2 v}{\partial x^2}\right)$$
$$+ \left(2\left(\frac{\partial u}{\partial y}\right)^2 + 2u\frac{\partial^2 u}{\partial y^2}\right) + \left(2\left(\frac{\partial v}{\partial y}\right)^2 + 2v\frac{\partial^2 v}{\partial y^2}\right)$$

$$= 2\left(\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right)$$
$$+ 2u\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + 2v\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

[By cauchy Riemann eq^n,

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \; ; \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

By laplace eq^n

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \; ; \quad \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$$]

$$= 2\left(\left(\frac{\partial u}{\partial x}\right)^2 + \left(+\frac{\partial v}{\partial x}\right)^2 + \left(-\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2\right) + 2u(0) + 2v(0)$$

$$= 2\left(2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial x}\right)^2\right) = 4\left(\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2\right)$$

$$= 4\,(u'^2 + v'^2)$$
$$= 4\left(\sqrt{u'^2+v'^2}\right)^2.$$
$$= \boxed{4|f'(z)|^2 = RHS}$$

$$\left[\begin{array}{l} f(z) = u + iv \\ f'(z) = u' + iv' \end{array}\right]$$

Hence proved

Q8. Show that

$$f(z) = \begin{cases} \dfrac{xy^2(x+iy)}{x^4+y^4} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$$

is not analytic at $z=0$ although CR equations are satisfied at origin

$$f(z) = \frac{xy^2(x+iy)}{x^4+y^4} = \frac{x^2y^2}{x^4+y^4} + i\,\frac{xy^3}{x^4+y^4}$$

$$f(z) = u(x,y) + iv(x,y)$$

$\therefore\ u(0,0) = 0\,;\ v(0,0) = 0$ (at $z=0$, ie $x, y=0$)

$$\left(\frac{\partial u}{\partial x}\right)_{(0,0)} = \lim_{x\to 0} \frac{u(x,0)-u(0,0)}{x}$$

$$= \lim_{x\to 0} \frac{x^2 \times 0/x^4+0^4 - 0}{x} = 0$$

$$\left(\frac{\partial u}{\partial y}\right)_{(0,0)} = \lim_{y\to 0} \frac{u(0,y)-u(0,0)}{y}$$

$$= \lim_{y\to 0} \frac{0\times y^2/x^4+0 - 0}{y} = 0$$

$$\left(\frac{\partial v}{\partial x}\right)_{(0,0)} = \lim_{x\to 0} \frac{v(x,0)-v(0,0)}{x} = 0$$

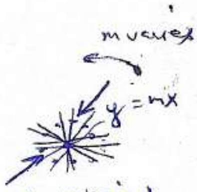
$$\left(\frac{\partial v}{\partial y}\right)_{(0,0)} = \lim_{y\to 0} \frac{v(0,y)-v(0,0)}{y} = 0$$

hence at $(0,0)$ CR equations are satisfied

ie $\dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y} = 0\ ;\ \dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x} = 0$

$\boxed{\text{BUT}}$ $$f'(0) = \lim_{z\to 0} \frac{f(z)-f(0)}{z} = \lim_{z\to 0} \frac{f(z)}{z}$$

$$= \lim_{z\to 0} \frac{xy^2(x+iy)}{x^4+y^4}$$



if $z \to 0$ along path $y = mx$ (ie from all pts in vicinity of origin along lines)

then $$f'(0) = \frac{m^2(1+im)}{1+m^4}$$

$\boxed{\text{WHICH ASSUMES DIFFERENT VALUES AS } m \text{ VARIES} \ \therefore f'(0) \text{ is not UNIQUE at } (0,0)}$

Thus, $f(z)$ is not analytic at origin even ~~though~~ it ~~is continuous &~~ satisfies CR equations.

Q9. Solve the differential equation

$$ty'' + (1-2t)y' - 2y = 0 \quad \text{when } y(0) = 1 \; ; \; y'(0) = 2$$

using laplace transform $\qquad$ let $\mathcal{L}\{y(t)\} = Y(s)$

Taking laplace transform on both sides

$$\mathcal{L}\{ty''\} + \mathcal{L}\{y'\} - 2\mathcal{L}\{ty'\} - 2\mathcal{L}\{y\} = 0$$

$$(-1)^1 \frac{d}{ds}\left(s^2 Y - s\overset{1}{y(0)} - \overset{2}{y'(0)}\right) + sY - y(0)$$
$$- 2(-1)^1 \frac{d}{ds}(sY - y(0)) - 2Y = 0$$

$$-\left[2sY + s^2Y' - \cancel{1}\right] + sY - 1 + 2\left[Y + sY'\right] - 2Y = 0$$

$$-2sY - s^2Y' + \cancel{1} + sY - \cancel{1} + 2Y + 2sY' - 2Y = 0$$

$$(2s - s^2)Y' + (s - 2s + \cancel{2} - \cancel{2})Y = 0$$

$$(2\cancel{s} - s^{\cancel{2}})Y' - \cancel{s}Y = 0$$

$$(2 - s)Y' - Y = 0.$$

$$(2 - s)\frac{dY}{ds} = Y.$$

$$\int \frac{1}{Y}\,dY = \int \frac{1}{2-s}\,ds$$

$$\ln Y = \ln\left(c/s-2\right)$$

$$Y = c/s-2$$

Taking inverse laplace

$$\mathcal{L}^{-1}\{Y\} = \mathcal{L}^{-1}\{c/s-2\}$$

$$y(t) = ce^{2t}$$

$$y(0) = 1 = ce^0 \rightsquigarrow c = 1$$

Sol^n. $\therefore \boxed{y(t) = e^{2t}}$

Q 10 Express the given function in terms of unit step function

$$f(t) = \begin{cases} \cos t & 0 < t < \pi \\ 1 & \pi < t < 2\pi . \\ \sin t & t > 2\pi \end{cases}$$

and hence find its laplace transform

$$u(t-a) = \begin{cases} 0 & t < a \\ 1 & t \geq a \end{cases}$$

$$f(t) = [u(t-0) - u(t-\pi)]\cos t + [u(t-\pi) - u(t-2\pi)]\, 1 + [u(t-2\pi) - u(\cancel{t-\infty})]\sin t .$$

0

$$= u(t-0)\cos t + u(t-\pi)[-\cos t + 1] \; u(t-2\pi)[-1 + \sin t]$$

$$\left(\begin{matrix}\cos t-\pi = -\cos t \\ \sin t - 2\pi = \sin t\end{matrix}\right)$$

~~$\mathcal{L}\{f(t)\}$~~

$$f(t) = u(t-0)\cos(t-0) + u(t-\pi)\cdot\cos(t-\pi) + u(t-2\pi)\sin(t-2\pi) + u(t-\pi) - u(t-2\pi)$$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{u(t-0)\cos(t-0)\} + \mathcal{L}\{u(t-\pi)\cos(t-\pi)\} + \mathcal{L}\{u(t-2\pi)\sin(t-2\pi\} + \mathcal{L}\{u(t-\pi)\}$$
$$\pm \mathcal{L}\{u(t-2\pi)\} .$$

By second shifting theorem ;

$$\mathcal{L}\{f(t-a)\,u(t-a)\} = e^{-as}F(s)$$

& $\mathcal{L}\{u(t-a)\} = e^{-sa}/s$

$$\mathcal{L}\{f(t)\} = e^{-0s}\left[\frac{s}{1+s^2}\right] + e^{-\pi s}\left[\frac{s}{1+s^2}\right]$$

$$+ e^{-2\pi s}\left[{}^{1}/_{1+s^2}\right] + \frac{e^{-s\pi}}{-s} \pm \frac{e^{-2s\pi}}{-s}$$

$$\boxed{\mathcal{L}\{f(t)\} = \frac{s + s e^{-\pi s} + e^{-2\pi s}}{1+s^2} \pm \frac{e^{-s\pi}}{s}\left[1 - e^2\right]}$$

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