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ASSIGNMENT-2

ENGINEERING MATHEMATICS

Submitted by : YASH VINAYVANSHI
B.TECH 2nd SEM
SECTION C-72.
R.NO.: 19BCS081
JAMIA MILLIA ISLAMIA
Email: yash.vinayvanshi@gmail.com

Submitted to : PROF. IDRIS QURESHI
PROF. SAIMA
PROF. NAVED AKHTAR
DEPT. OF APPLIED SCIENCES
JMI FET.

Q1. Find power series soln for the given differential eqn.

$$(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$$

comparing with $P(x)\frac{d^2y}{dx^2} + Q(x)\frac{dy}{dx} + R(x) = 0$

$P(x) = 1+x^2$; $Q(x) = x$; $R(x) = -1$

at $x=0$; $P(0) = 1+0^2 = 1 \neq 0$ ⇝ $x=0$ is ordinary pt.

∴ Assuming soln $y = \sum_{r=0}^{\infty} a_r x^r$

and this should satisfy the diffn eqn.

$$\frac{dy}{dx} = \sum_{r=0}^{\infty} a_r r x^{r-1}$$

$$\frac{d^2y}{dx^2} = \sum_{r=0}^{\infty} a_r r(r-1) x^{r-2}$$

$$\therefore (1+x^2)\sum a_r r(r-1)x^{r-2} + x\sum a_r r x^{r-1} - \sum a_r x^r = 0$$

$$\cancel{\sum a_r + \sum a_r r(r-1)x} \sum a_r (r)(r-1)x^{r-2} + \sum a_r (r)(r-1)x^r + \sum a_r r x^r - \sum a_r x^r = 0$$

$$\sum a_r r(r-1) x^{r-2} + \sum a_r [r(r-1) + r - 1] x^r = 0$$

$\{$put $r = r+2$

$$\sum a_{r+2}(r+2)(r+1)x^r + \sum a_r [r^2 - \cancel{r} + \cancel{r} - 1] x^r = 0$$

$$\sum (a_{(r+2)}[(r+2)(r+1)] + a_r[r^2-1]) x^r = 0$$

if series $= 0 \Rightarrow$ coefficients $= 0$

$$a_{r+2}(r+2)(r+1) + a_r(r^2-1) = 0.$$

$$a_{r+2} = \frac{1-r^2}{(r+1)(r+2)}\, a_r$$ Recurrence relation

$r=0 \rightsquigarrow \quad a_2 = \frac{1-0}{(0+1)(0+2)}\, a_0 \Rightarrow \frac{1}{2}a_0$

$r=1 \rightsquigarrow \quad a_3 = \frac{1-1^2}{(1+1)(1+2)}\, a_1 \Rightarrow 0$

$r=2 \rightsquigarrow \quad a_4 = \frac{1-2^2}{(1+2)(2+2)}\, a_2 = \frac{-3}{3\times 4}\, a_2 = -\frac{1}{4}\, a_2 = -\frac{1}{8}\, a_0$

$r=3 \rightsquigarrow \quad a_5 = \frac{1-9}{(4)(5)}\, a_3 = \frac{-8}{20}\, a_3 = 0$

$r=4 \rightsquigarrow \quad a_6 = \frac{1-16}{5(6)}\, a_4 = \frac{-15}{30}\, a_4 = -\frac{1}{2}\, a_4 = \frac{1}{16}\, a_0$

$r=5 \rightsquigarrow \quad a_7 = \frac{1-25}{6\times 7} = \frac{-24}{42}\, a_5 = -\frac{1}{2}\, a_5 = 0$

$r=6 \rightsquigarrow \quad a_8 = \frac{1-36}{7\times 8} = \frac{\cancel{-35}^{\,5}}{\cancel{56}_{\,8}}\, a_6 = -\frac{5}{8} \times \frac{1}{16}\, a_0 = -\frac{5}{128}\, a_0$

$$\therefore \quad \boxed{y = a_0\left(1 + \frac{x^2}{2} - \frac{x^4}{8} + \frac{x^6}{16} - \frac{5x^8}{128} + \cdots\right) + a_1 x}$$

$\int \frac{\cos x}{\sin x} =$

$\sin x = t$

Q2 Solve the following LPDE using lagrange's method

(a) $(mz - ny)\frac{\partial z}{\partial x} + (nx - lz)\frac{\partial z}{\partial y} = ly - mx$

(b) $\tan x \frac{\partial z}{\partial x} + \tan y \frac{\partial z}{\partial y} = \tan z$

(a) $$\frac{dx}{mz-ny} = \frac{dy}{nx-lz} = \frac{dz}{ly-mx}$$ (subsidiary eqns)

$$\Rightarrow \frac{x\,dx + y\,dy + z\,dz}{x(mz-ny) + y(nx-lz) + z(ly-mx)} = \lambda$$

$$= \frac{x\,dx + y\,dy + z\,dz}{\cancel{mx}z - n\cancel{xy} + y\cancel{nx} - y\cancel{lz} + z\cancel{ly} - \cancel{mxz}} = \lambda$$

$$= \frac{x\,dx + y\,dy + z\,dz}{0} = \lambda$$

$$\int x\,dx + y\,dy + z\,dx = \int 0$$

$$\frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} = c_1 \sim \boxed{x^2 + y^2 + z^2 = a}$$

Also,

$$\Rightarrow \frac{l\,dx + m\,dy + n\,dz}{l(mz-ny) + m(nx-lz) + n(ly-mx)} = \lambda$$

$$\Rightarrow \frac{l\,dx + m\,dy + n\,dz}{lm\cancel{z} - l\cancel{ny} + m\cancel{nx} - \cancel{ml}z + nly - \cancel{nmx}} = \lambda$$

$$\Rightarrow \frac{l\,dx + m\,dy + n\,dz}{0} = \lambda.$$

$$\Rightarrow \int l\,dx + m\,dy + n\,dz = \int 0$$

$$\boxed{lx + my + nz = a}$$

$\therefore$ General solⁿ is $\boxed{\phi(x^2+y^2+z^2,\ lx+my+nz) = 0.}$

(b) $$\frac{dx}{\tan x} = \frac{dy}{\tan y} = \frac{dz}{\tan z}$$ (subsidiary eqns)

$\int \cot x\,dx = \int \cot y\,dy$; $\int \cot x\,dx = \int \cot z\,dz$

$\ln \sin x = \ln \sin ay$; $\ln \sin x = \ln \sin bz$

$\sin x = \sin ay$; $\sin x = \sin bz.$

$\therefore$ General solⁿ is $\boxed{\phi(\sin x - \sin ay,\ \sin x - \sin bz) = 0}$

Q3. Solve $z^2(p^2x^2+q^2) = 1$

if $f(p,q,z) = 0$

Assume sol^n as z is a $func^n$ of $x+ay = u$.

$$p = \frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} = \frac{dz}{du} \quad ; \quad q = \frac{\partial z}{\partial y} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y} = a\frac{dz}{du}$$

$f\left(z, \frac{dz}{du}, a\frac{dz}{dy}\right) = 0$ is ODE of first order

$\therefore \int \frac{dz}{du} = \int \phi(z, a)$ to get complete sol^n

$$z^2\left[\left(x\frac{\partial z}{\partial x}\right)^2 + \frac{\partial z}{\partial u}\right] = 1.$$

Let $x = \log X \quad \therefore \quad x\frac{\partial z}{\partial x} = \frac{\partial z}{\partial X}$

$$z^2\left[\left(\frac{\partial z}{\partial X}\right)^2 + \frac{\partial z}{\partial u}\right] = 1$$

Let $u = X + ay \rightsquigarrow \frac{\partial z}{\partial X} = \frac{dz}{du} \;;\; \frac{\partial z}{\partial y} = a\frac{dz}{du}.$

$$z^2\left[\left(\frac{dz}{du}\right)^2 + a^2\left(\frac{dz}{du}\right)^2\right] = 1$$

$$z^2(1+a^2)\left(\frac{dz}{du}\right)^2 = 1.$$

$$\pm z(1+a^2)^{1/2}\frac{dz}{du} = 1$$

$$\int (1+a^2)^{1/2} z\,dz = \int \pm du$$

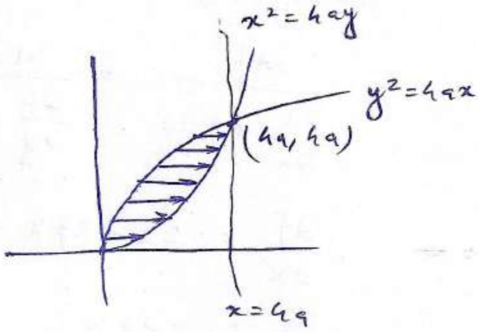
$$(1+a^2)^{1/2}\frac{z^2}{2} = \pm u + b$$

$$z^2 = \frac{\pm 2\overset{\log X + ay}{\cancel{u}} + b}{(1+a^2)^{1/2}}$$

$$\boxed{z^2 = \frac{\pm 2(\log X + ay) + b)}{(1+a^2)^{1/2}}}$$

Q5. Evaluate

$$\int_{x=0}^{4a}\int_{y=x^2/4a}^{2\sqrt{ax}} dy\,dx \quad \text{by change of order of integration}$$



	x	y
Lower limit	0	$y = x^2/4a \rightsquigarrow x^2 = 4ay$
upper limit	$4a$	$y = 2\sqrt{ax} \rightsquigarrow y^2 = 4ax$

Change of order

upper limit $x = 2\sqrt{ay}$ — 0

lower limit $y^2 = 4ax \to x = y^2/4a$ — $4a$

$$= \int_{y=0}^{4a}\int_{x=y^2/4a}^{x=2\sqrt{ay}} dx\,dy$$

$$= \int_{y=0}^{4a} \left[x\right]_{y^2/4a}^{2\sqrt{ay}} dy$$

$$= \int_{y=0}^{4a} \left(2\sqrt{ay} - y^2/4a\right) dy$$

$$= \left[2\sqrt{a}\; y^{3/2} \times \frac{2}{3}\right]_0^{4a} - \frac{1}{4a}\left[\frac{y^3}{3}\right]_0^{4a}$$

$$= \frac{4\sqrt{a}}{3}\left[(4a)^{3/2}\right] - \frac{1}{\cancel{4a}}\,\frac{(4a)^{\cancel{3}}}{3}$$

$$= \frac{32a^2}{3} - \frac{16a^2}{3}$$

$$= \boxed{\frac{16a^2}{3}}$$

Q4. solve $2xz - px^2 - 2qxy + pq = 0$ ⑥

$$f(x, y, z, p, q) = 2xz - px^2 - 2qxy + pq = 0$$

charpit's subsidiary eq^ns.

$$\frac{dp}{\frac{\partial f}{\partial x} + p\frac{\partial f}{\partial z}} = \frac{dq}{\frac{\partial f}{\partial y} + q\frac{\partial f}{\partial z}} = \frac{dz}{-p\frac{\partial f}{\partial p} - q\frac{\partial f}{\partial q}} = \frac{dx}{-\frac{\partial f}{\partial p}} = \frac{dy}{-\partial f/\partial q}$$

$$\frac{\partial f}{\partial x} = 2z - 2px - 2qy \quad ; \quad \frac{\partial f}{\partial z} = 2x$$

$$\frac{\partial f}{\partial y} = -2qx \quad ; \quad \frac{\partial f}{\partial p} = -x^2 + q \quad ; \quad \frac{\partial f}{\partial q} = -2xy + p$$

$$\frac{dp}{2z - \cancel{2px} - 2qy + \cancel{2px}} = \frac{dq}{-\cancel{2qx} + \cancel{2qx}} = \frac{dz}{px^2 - pq + 2qxy - pq}$$

$$= \frac{dx}{-(-x^2+q)} = \frac{dy}{-(-2xy+p)}$$

$$\frac{dp}{2z - 2qy} = \frac{dq}{0} = \frac{dz}{px^2 - 2pq + 2qxy} = \frac{dx}{x^2 - q} = \frac{dy}{2xy - p} = \lambda$$

$$\frac{dq}{0} = \lambda \rightsquigarrow \int dq = \int 0 \rightsquigarrow q = a$$

$$2xz - px^2 - 2axy + pq = 0$$

$$p(-x^2 + q) + 2xz - 2qxy = 0$$

$$p = \frac{2x(z - ay)}{x^2 - a}$$

by total derivative,

$$dz = p\,dx + q\,dy$$

$$dz = \frac{2x(z-ay)}{x^2-a}dx + a\,dy$$

$$\frac{dz - a\,dy}{z - ay} = \frac{2x\,dx}{x^2 - a}$$

$$(z - ay = t \rightsquigarrow dz - a\,dy = dt)$$

$$\log(z - ay) = \log(x^2 - a) + \log b$$

$$z - ay = b(x^2 - a)$$

$$\boxed{z = ay + b(x^2 - a)}$$

Q7. Evaluate

$$\int_0^{\log 2}\int_0^{x}\int_0^{x+\log y} e^{x+y+z}\,dx\,dy\,dz$$

$$\int_{x=0}^{\log 2}\int_{y=0}^{x}\int_{z=0}^{x+\log y} e^{x+y+z}\,dx\,dy\,dz$$

$$\int_{x=0}^{\log 2}\int_{y=0}^{x} e^{x}e^{y}\cdot\left[e^{z}\right]_0^{x+\log y}\,dy\,dx$$

$$\int_{x=0}^{\log 2}\int_{y=0}^{x} e^{x}e^{y}\left(e^{x+\log y}-1\right)dy\,dx$$

$$e^{x}e^{\log y}=e^{x}y$$

$$\cancel{\int_{x=0}^{\log 2}\int_{y=0}^{x} e^{2x}e^{y}\cdot y\,dy\,dx}$$

$$\cancel{\int_{x=0}^{\log 2} e^{2x}\int_{y=0}^{x}(e^{y}\cdot y)\,dy\,dx}$$

$$\cancel{\int_{x=0}^{\log 2} e^{2x}\left[y\int e^{y}-\int\frac{d}{dy}y\int e^{y}\right]_0^{x}dx}$$

$$\cancel{=\left[ye^{y}-\int\int e^{y}\right]_0^{x}}$$

$$\cancel{=\left[ye^{y}-e^{y}\right]_0^{x}}$$

$$\cancel{\int_{x=0}^{\log 2} e^{2x}\; xe^{x}-e^{x}}$$

$$=\int_{x=0}^{\log 2}\int_0^{x} e^{2x}e^{y}y-e^{x}e^{y}\,dy\,dx$$

$$=\int_0^{\log 2}\int_0^{x} e^{2x}e^{y}y\,dy\,dx-\int_0^{\log 2}\int_0^{x} e^{x}e^{y}\,dy\,dx$$

$$=\int_0^{\log 2} e^{2x}\left[ye^{y}-\int\frac{d}{dy}y\int e^{y}\right]_0^{x}dx-\int_0^{\log 2} e^{x}\left[e^{y}\right]_0^{x}$$

$$=\int_0^{\log 2} e^{2x}\left[xe^{x}-e^{x}-1\right]dx-\int_0^{\log 2} e^{x}(e^{x}-1)\,dx$$

$$=\int_0^{\log 2} xe^{3x}-e^{3x}-e^{2x}\,dx-\int_0^{\log 2} e^{2x}-e^{x}\,dx$$

$$\left[\frac{x}{3}e^{3x}-\frac{1}{9}e^{3x}-\frac{e^{3x}}{3}-\frac{e^{2x}}{2}\right]_0^{\log 2}-\left[\frac{e^{2x}}{2}-e^{x}\right]_0^{\log 2}$$

$$\frac{\log 2}{3}e^{\log 2^3}-\frac{1}{9}e^{\log 2^3}-\frac{e^{\log 2^3}}{3}-\frac{e^{\log 2^2}}{2}+\frac{1}{9}+\frac{1}{3}+\frac{1}{2}-\left(\frac{e^{\log 4}}{2}-e^{\log 2}\cancel{-1}\cancel{+1}\right)$$

$$\frac{8\log 2}{3}-\frac{8}{9}-\frac{8}{3}-2+\frac{1}{9}+\frac{1}{3}+\frac{1}{2}\cancel{-2}\cancel{+2}$$

$$\frac{8\log 2}{3}-\frac{17}{6}=\boxed{\frac{16\log 2-17}{6}}$$

Q6 Evaluate $\iint_D xy\sqrt{1-x-y}\,dx\,dy$ where D is the region bounded by $x=0$, $y=0$ and $x+y=1$, using transformation $x+y=u$, $y=uv$

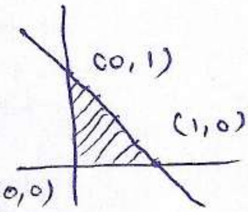
$$x+y=u \quad ; \quad y=uv \qquad -\ (1)$$

$$x=u-y \rightsquigarrow x=u-uv \qquad -\ (2)$$

$$J\ \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \partial x/\partial u & \partial x/\partial v \\ \partial y/\partial u & \partial y/\partial v \end{vmatrix} = \begin{vmatrix} 1-v & v \\ -u & u \end{vmatrix}$$

$$= (1-v)u-(-uv)$$

$$= u-uv+uv = u$$



using boundary conditions to fix limits.

$$\left.\begin{array}{l} x=0,\ y=0 \rightarrow u=0 \\ x=1,\ y=0 \rightarrow u=1 \end{array}\right] : u: 0\rightarrow 1$$

$$\left.\begin{array}{l} x=0,\ y=0 \rightsquigarrow v=0 \\ x=1,\ y=1 \rightsquigarrow v=1 \end{array}\right] \ v: 0\rightarrow 1.$$

$$\iint_D xy\sqrt{1-x-y}\,dx\,dy = \int_0^1\int_0^1 u(1-v)\,uv\sqrt{1-u}\cdot|\cancel{J}|^{u}\,du\,dv$$

$$= \int_0^1\int_0^1 u^3(1-u)^{1/2}\,v(1-v)\,du\,dv$$

$$= \int_0^1 v(1-v)\,dv \int_0^1 u^3(1-u)^{1/2}\,du$$

Let $u=\sin^2\theta \rightsquigarrow du = 2\sin\theta\cos\theta\,d\theta$

$$= \left[\frac{v^2}{2}-\frac{v^3}{3}\right]_0^1 \times \int_0^{\pi/2} \sin^6\theta\,(\cancel{1-\sin^2\theta})^{1/2}_{\cos^2\theta}\ 2\sin\theta\cos\theta\,d\theta$$

$$= \left(\frac{1}{2}-\frac{1}{3}\right)\times 2\int_0^{\pi/2}\sin^7\theta\cos^2\theta\,d\theta$$

$$= \frac{2}{6}\times\frac{6\cdot 1}{9\cdot 7\cdot 5\cdot 3} = \boxed{\frac{2}{945}}$$

$$\left(\text{ie } \int_0^{\pi/2}\sin^m x\cos^n x\,dx = \frac{(m-1)(m-3)\times\ldots(n-1)\times(n-3)\ldots}{(m+n)(m+n-2)(m+n-4)\ldots}\right.$$

$$\left.\text{if not both } m, n \text{ even}\right)$$

Q8. solve by means of Frobenius method.

$$2x(1-x)y'' + (1-x)y' + 3y = 0$$

Let $y = x^m \rightsquigarrow \dfrac{dy}{dx} = mx^{m-1}$; $\dfrac{d^2y}{dx^2} = m(m-1)x^{m-2}$

$$(2x - 2x^2)\, m(m-1)x^{m-2} + (1-x)\, mx^{m-1} + 3x^m = 0$$

$$2m(m-1)x^{m-1} - 2m(m-1)x^m + mx^{m-1} - mx^m + 3x^m = 0$$

$$x^{m-1}\left[2m(m-1) + m\right] + x^m\left[-2m(m-1) - m + 3\right] = 0$$

Highest degree - lowest degree $= m - (m-1) = 1$.

indice eqⁿ

$2m^2 - 2m + m = 0$

$2m^2 - m = 0$

$m(2m-1) = 0$

$m = 0$; $m = 1/2$

roots are distinct and do not differ by integer

~~solⁿ is of form~~

$\therefore$ assume $y = \sum_{r=0}^{\infty} a_r x^{m+r}$

$$\therefore \frac{dy}{dx} = \sum a_r (m+r)x^{m+r-1}$$

$$\frac{d^2y}{dx^2} = \sum a_r (m+r)(m+r-1)x^{m+r-2}$$

This should satisfy the eqⁿ

$$2x(1-x)\left[\sum a_r (m+r)(m+r-1)x^{m+r-2}\right] + (1-x)\left[\sum a_r (m+r)x^{m+r-1}\right] + 3\sum a_r x^{m+r} = 0$$

$$\sum a_r x^{m+r}\left[-2(m+r)(m+r-1) - m+r + 3\right] + \sum a_r x^{m+r-1}\left[2(m+r)(m+r-1) + (m+r)\right]$$

} put $r = r+1$ in second term.

$$\sum a_r x^{m+r}\left[(m+r)(-2m-2r+2-1) + 3\right] + \sum a_{r+1} x^{m+r}\left[2(m+r)(m+r+1) + (m+r+1)\right]$$

series $= 0 \rightsquigarrow$ coefficient $= 0$.

$$a_r\left[(m+r)(-2m-2r+1) + 3\right] + a_{r+1}\left[(m+r+1)(2m+2r+1)\right] = 0$$

$$\cancel{a_r = -a_{r+1}\frac{\left[(m+r+1)(2m+2r+1)\right]}{\left[(m+r)(-2m-2r+1) + 3\right]}}$$

$$a_{r+1} = -a_r \frac{\left[(m+r)(-2m-2r+1) + 3\right]}{(m+r+1)(2m+2r+1)}$$

$$\therefore \quad a_{r+1} = \frac{(m+r)(2m+2r-1)-3}{(m+r+1)(2m+2r+1)}\, a_r$$

$$r=0 \rightsquigarrow a_1 = \frac{m(2m-1)-3}{(m+1)(2m+1)}\, a_0$$

$$= \frac{2m^2-m-3}{(m+1)(2m+1)}\, a_0$$

$$= \frac{(m-3/2)\cancel{(m+1)}}{\cancel{(m+1)}(2m+1)}\, a_0 = \boxed{\frac{m-3/2}{2m+1}\, a_0}$$

$$r=1 \rightsquigarrow a_2 = \frac{(m+1)(2m+2-1)-3}{(m+2)(2m+2+1)}\, a_1$$

$$= \frac{(m+1)(2m+1)-3}{(m+2)(2m+3)}\, a_1$$

$$= \frac{(m-1/2)\cancel{(m+2)}}{\cancel{(m+2)}(2m+3)}\, a_1 = \frac{(m-1/2)}{(2m+3)}\, a_1$$

$$= \boxed{\frac{(m-1/2)(m-3/2)}{(2m+3)(2m+1)}\, a_0}$$

$$r=2 \rightsquigarrow a_3 = \frac{(m+2)(2m+4-1)-3}{(m+3)(2m+4+1)}\, a_2$$

$$= \frac{(m+2)(2m+3)-3}{(m+3)(2m+5)}\, a_2$$

$$= \frac{(m+1/2)\cancel{(m+3)}}{\cancel{(m+3)}(2m+5)}\, a_2 = \boxed{\frac{(m+1/2)(m-1/2)(m-3/2)}{(2m+5)(2m+3)(2m+1)}\, a_0}$$

$$y = a_0 x^m + a_1 x^{m+1} + a_2 x^{m+2} + a_3 x^{m+3} + \cdots$$

$$y = a_0 x^m \left[1 + \frac{m-3/2}{2m+1}x + \frac{(m-1/2)(m-3/2)}{(2m+3)(2m+1)}x^2 + \frac{(m+1/2)(m-1/2)(m-3/2)}{(2m+5)(2m+3)(2m+1)}x^3 + \cdots\right]$$

at $m=0$,

$$y = a_0\left[1 - \frac{3}{2}x + \frac{(-1/2)(-3/2)}{3\times 1}x^2 + \frac{1/2(-1/2)(-3/2)}{5\times 3\times 1}x^3 + \cdots\right]$$

$$= a_0\left[1 - 3/2\,x + 1/4\,x^2 - 1/40\,x^3 + \cdots\right]$$

at $m = 1/2$

$$y = a_0\left[1 + \frac{1/2-3/2}{1+1}x + \frac{(1/2-1/2)(1/2-3/2)}{(1+3)(1+1)}x^2 + \frac{(1/4)(0)(1/2-3/2)}{6\times 4\times 2}\right]$$

$$= a_0\left[1 - 3/4\,x + 0 + 0 + \cdots\right]$$

Sol^n is $y = A(y)_{m=0} + B(y)_{m=1/2} = \boxed{A\left[1 - 3/4\,x\right] + B\left[1 - 3/2\,x + 1/4\,x^2 - 1/40\,x^3 + \cdots\right]}$

Q9. Solve the ODE in series

$$x^2 y'' + x(x-1)y' + (1-x)y = 0 \text{ about } x=0$$

compare with $P(x)y'' + Q(x)y' + R(x)y = 0$.

at $x=0$, $P(0) = 0^2 = 0 \rightsquigarrow$ singular pt.

$$\lim_{x\to 0} (x-0)\frac{x(x-1)}{x^2} = -1 \quad (\text{finite})$$

$$\lim_{x\to 0} (x-0)^2 \frac{1-x}{x^2} = 1 \quad (\text{finite}) \to \text{regular singular pt}.$$

$\therefore$ series soln exist and frobenius method can be used

$$y = x^m \rightsquigarrow \frac{dy}{dx} = mx^{m-1} \quad ; \quad \frac{d^2y}{dx^2} = m(m-1)x^{m-2}$$

$$\therefore x^2 m(m-1)x^{m-2} + x^2 m x^{m-1} - mx x^{m-1} + x^m - x x^m = 0$$

$$m(m-1)x^m + mx^{m+1} - mx^m + x^m - x^{m+1} = 0$$

$$x^{m+1}(m-1) + x^m(m^2 - m - m + 1) = 0$$

p = highest degree of x - lowest degree of x

$= m+1-m = 1$.

$$\boxed{\begin{array}{c}\text{indicial eq}^n \\ m^2 - 2m + 1 = 0 \\ (m-1)^2 = 0 \\ m = 1, 1\end{array}}$$

$$\therefore y = \sum_{r=0}^{\infty} a_r x^{m+pr} = \sum_{r=0}^{\infty} a_r x^{m+r}$$

$\therefore$ soln is

$$y = A y_{m_1} + B\left(\frac{\partial y}{\partial m}\right)_{m_1}$$

$$\therefore \frac{dy}{dx} = \sum a_r (m+r) x^{m+r-1}$$

$$\therefore \frac{d^2y}{dx^2} = \sum a_r (m+r)(m+r-1) x^{m+r-2}$$

This should satisfy the eqn

$$x^2 \sum a_r (m+r)(m+r-1) x^{m+r-2} + x^2 \sum a_r (m+r) x^{m+r-1}$$
$$- x \sum a_r (m+r) x^{m+r-1} + \sum a_r x^{m+r} + \sum a_r x^{m+r+1} = 0$$

$$\sum a_r [(m+r) + 1] x^{m+r+1} + \sum a_r [(m+r)(m+r-1) - (m+r) + 1] x^{m+r} = 0$$

$$\} \; r = r-1$$

$$\sum a_{r-1}[m+r-1+1] x^{m+r} + \sum a_r [(m+r)[m+r-1-1] + 1] x^{m+r} = 0$$

$$a_{r-1}[\cancel{m+r}] + a_r[(\cancel{m+r})\cancel{(m+r-2)+1}] = 0.$$

$$m+r-1.$$

$$a_{r-1} = a_r(m+r-1) \quad \text{recursive rel}^n.$$

$$r=1 \rightsquigarrow a_0 = a_1(m) \rightsquigarrow a_1 = a_0/m$$

$$r=2 \rightsquigarrow a_1 = a_2(m+1) \rightsquigarrow a_2 = a_1/m+1 = \frac{a_0}{m(m+1)}$$

$$r=3 \rightsquigarrow a_2 = a_3(m+2) \rightsquigarrow a_3 = a_2/m+2 = \frac{a_0}{m(m+1)(m+2)}$$

$$y = a_0 x^m + a_1 x^{m+1} + a_2 x^{m+2} + \cdots$$

$$y = a_0 x^m + \frac{a_0}{m} x^{m+1} + \frac{a_0}{m(m+1)} x^{m+2} + \frac{a_0}{m(m+1)(m+2)} x^{m+3} + \cdots$$

$$= a_0 x^m \left[1 + \frac{x}{m} + \frac{x^2}{m(m+1)} + \frac{x^3}{m(m+1)(m+2)} + \cdots \right]$$

$$y_{m=1} = a_0 x \left[1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots \right]$$

$$\frac{\partial y}{\partial m} = m a_0 x^{m-1} \left[1 + \frac{x}{m} + \frac{x^2}{m(m+1)} + \frac{x^3}{m(m+1)(m+2)} + \cdots \right]$$

$$+ a_0 x^m \left[1 - \frac{x}{m^2} + \left(\frac{d}{dm} m^{-1} (m+1)^{-1} + \frac{d}{dm} (m+1)^{-1} m^{-1} \right) x^2 + \cdots \right]$$

$$\frac{-1}{m^2(m+1)} - \frac{1}{m(m+1)^2}$$

$$\left(\frac{\partial y}{\partial m}\right)_{m=1} = 1 a_0 x^0 \left[1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots \right]$$

$$+ a_0 x^1 \left[1 - \frac{x}{1} - \frac{1}{2} x^2 - \frac{1}{4} x^2 + \cdots \right]$$

$$= a_0 \left[1 + x + x^2/2 + x^3/6 + \cdots \right]$$

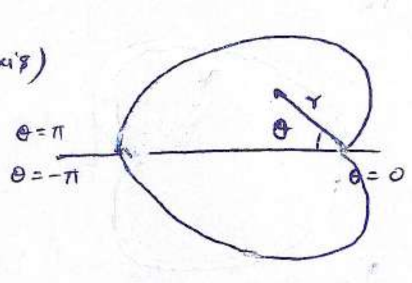
$$+ a_0 x \left[1 - x - \frac{3}{4} x^2 + \cdots \right]$$

sol^n is $y = A\, y_{m=1} + B \left(\frac{\partial y}{\partial m}\right)_{m=\phi}$

$$\boxed{\begin{aligned} y = {} & A \left[a_0 x \left[1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \cdots \right] \right] \\ & + B \left[a_0 \left[1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots \right] + a_0 x \left[1 - x - \frac{3}{4} x^2 + \cdots \right] \right] \end{aligned}}$$

Q10. Find by double integration, the centre of gravity of area of cardioid $r = a(1-\cos\theta)$

$$\bar{x} = \frac{\iint \rho r\cos\theta \cdot r\,d\theta\,dr}{\iint \rho r\,d\theta\,dr} \qquad \bar{y} = 0 \text{ (sym abt y axis)}$$



$$= \frac{\int_{-\pi}^{\pi}\int_0^{a(1-\cos\theta)} \cos\theta\, r^2\,dr\,d\theta}{\int_{-\pi}^{\pi}\int_0^{a(1-\cos\theta)} r\,dr\,d\theta}$$

$$= \frac{\int_{-\pi}^{\pi} \cos\theta \left|\frac{r^3}{3}\right|_0^{a(1-\cos\theta)} d\theta}{\int_{-\pi}^{\pi} \left|\frac{r^2}{2}\right|_0^{a(1-\cos\theta)} d\theta}$$

$$= \frac{2a}{3}\,\frac{\int_{-\pi}^{\pi} \cos\theta\,(1-\cos\theta)^3\,d\theta}{\int_{-\pi}^{\pi} (1-\cos\theta)^2\,d\theta}$$

$(1-\cos\theta)^3 = 1 - \cos^3\theta - 3\cos\theta + 3\cos^2\theta$

$(1-\cos\theta)^2 = 1 + \cos^2\theta - 2\cos\theta$

$\int_0^{\pi} \cos\theta\,d\theta = 0$

$\int_0^{\pi} \cos^3\theta\,d\theta = 0$.

$$= -\frac{2a}{3}\,\frac{2\int_0^{\pi} (3\cos^2\theta + \cos^4\theta)\,d\theta}{2\int_0^{\pi} (1+\cos^2\theta)\,d\theta}$$

$$\left\{ \int_{-\pi}^{\pi} \cos^n\theta\,d\theta = 2\int_0^{\pi} \cos^n\theta\,d\theta \quad n:\text{even} \atop = 0 \quad n:\text{odd} \right\}$$

$$= -\frac{2a}{3}\,\frac{\int_0^{\pi/2} (3\cos^2\theta + \cos^4\theta)\,d\theta}{2\int_0^{\pi/2} (1+\cos^2\theta)\,d\theta}$$

$$= -\frac{2a}{3} \cdot \frac{3\cdot\frac{1}{2}\cdot\frac{\pi}{2} + \frac{3\cdot 1}{4\cdot 2}\cdot\frac{\pi}{2}}{\frac{\pi}{2} + \frac{1}{2}\cdot\frac{\pi}{2}} = -\frac{5a}{6}$$

$\therefore$ centre of gravity of cardioid $= \boxed{(-5a/6,\ 0)}$.

YASH VINAYVANSHI

— x — x —