DATED : 27/04/20

ASSIGNMENT-5
ENGINEERING MATHEMATICS

Submitted by : YASH VINAYVANSHI
B. TECH 2^{nd} SEM
SECTION C-70
ROLL. NO. 19BCS081
JAMIA MILLIA ISLAMIA
Email : yash.vinayvanshi@gmail.com

Submitted to : PROF. IDRIS QURESHI
PROF. NAVED AKHTAR
PROF. SAIMA
DEPT. OF APPLEED SCIENCES
JMI FET.

Q1. Solve the PDE using method of multipliers.

$$(x^2+y^2)p + 2xyq = (x+y)z$$

Lagrange's subsidiary equation

$$\frac{dx}{x^2+y^2} = \frac{dy}{2xy} = \frac{dz}{(x+y)z}$$

$$\frac{dx}{dy} = \frac{x^2+y^2}{2xy} = \frac{x}{2y} + \frac{y}{2x}$$

Let $x/y = v \rightarrow x = vy \sim dx/dy = v + x\,dv/dy$.

$$v + x\frac{dv}{dx} = \frac{v}{2} + \frac{1}{2v}$$

$$x\frac{dv}{dx} = \frac{1}{2v} - \frac{v}{2} = \frac{1}{2v}(1-v^2)$$

$$\int \frac{2v}{1-v^2}\,dv = \int \frac{1}{x}\,dx$$

$$-\log(1-v^2) = \log cy.$$

$$1 - v^2 = \frac{c}{y}$$

$$1 - \frac{x^2}{y^2} = \frac{c}{y} \rightsquigarrow \frac{y^2 - x^2}{y^{\cancel{2}}} = \frac{c}{\cancel{y}}$$

$$\boxed{y^2 - x^2 - cy = 0}$$

Also, $$\frac{dx + dy}{x^2+y^2+2xy} = \frac{dz}{(x+y)z}$$

$$\int \frac{dx+dy}{(x+y)^{\cancel{2}}} = \int \frac{dz}{\cancel{(x+y)}z}$$

$$\log(x+y) = \log cz.$$

$$\boxed{x+y = cz}$$

$\therefore$ General solⁿ is

$$\boxed{f(y^2 - x^2 - cy,\ x+y-cz) = 0}$$

Q2 Solve the non-linear PDE

$$z^2(p^2+q^2) = x^2+y^2$$

$$\left(z\frac{\partial z}{\partial x}\right)^2 + \left(z\frac{\partial z}{\partial y}\right)^2 = x^2+y^2$$

Put $\quad z\,dz = dZ \rightsquigarrow \int z\,dz = \int dZ \rightsquigarrow Z = z^2/2$

$$\frac{\partial Z}{\partial x} = \frac{\partial Z}{\partial z} \cdot \frac{\partial z}{\partial x} = z\frac{\partial z}{\partial x} = P$$

$$\frac{\partial Z}{\partial y} = \frac{\partial Z}{\partial z} \times \frac{\partial z}{\partial y} = z\frac{\partial z}{\partial y} = Q$$

$$\therefore \quad P^2 + Q^2 = x^2+y^2$$

$$P^2 - x^2 = y^2 - Q^2 = c \quad \text{say.}$$

$$p = \sqrt{x^2+c} \quad ; \quad q = \sqrt{y^2-c}$$

$$dz = p\,dx + q\,dy$$

$$dz = \sqrt{x^2+c}\,dx + \sqrt{y^2-c}\,dy$$

$$z = \frac{1}{2} x\sqrt{x^2+c} + \frac{1}{2} a \log[x + \sqrt{x^2+c}]$$

$$+ \frac{1}{2} y\sqrt{y^2 \cancel{-c}} - \frac{1}{2} a \log[y + \sqrt{y^2-c}] + b.$$

$$\frac{z^2}{2} = \frac{1}{2} x\sqrt{x^2+c} + \frac{1}{2} y\sqrt{y^2-c} + \frac{1}{2} c \log \frac{x + \sqrt{x^2+c}}{y + \sqrt{y^2-c}} + b$$

$$\boxed{z = x\sqrt{x^2+c} + y\sqrt{y^2-c} + c \log \frac{x + \sqrt{x^2+c}}{y + \sqrt{y^2-c}} + 2b}$$

Q3. Solve the differential epuation in series

$$(1-x^2)y'' - xy' + 4y = 0$$

$x = 0 \rightsquigarrow P(0) = 1 - 0^2 = 1 \neq 0 \quad \therefore$ ordinary pt.

$$\therefore \quad y = \sum_{r=0}^{\infty} a_r x^r \; ; \; \frac{dy}{dx} = \sum_{r=0}^{\infty} a_r r x^{r-1} \; ; \; \frac{d^2y}{dx^2} = \sum_{r=0}^{\infty} a_r r(r-1) x^{r-2}$$

This should satisfy the epuation,

$$(1-x^2)\sum a_r r(r-1)x^{r-2} - x\sum a_r r x^{r-1} + 4\sum a_r x^r = 0$$

$$\sum a_r r(r-1)x^{r-2} - \sum a_r r(r-1)x^r - \sum a_r r x^r + 4\sum a_r x^r = 0$$

$$\sum a_r[-r(r-1) - r + 4] + \underbrace{\sum a_r r(r-1)x^{r-2}}_{r = r+2} = 0$$

$$\sum a_r[-r^2 + \cancel{r} - r + 4] + \sum a_{r+2}(r+2)(r+1)x^r = 0$$

series $= 0 \rightsquigarrow$ coefficient $= 0$

$$a_r(-r^2+4) + a_{r+2}(r+2)(r+1) = 0$$

$$a_{r+2} = \frac{\cancel{r^2-4}\,(r-2)(r+2)}{\cancel{(r+2)}(r+1)} = \frac{r-2}{r+1}a_r$$

$$a_{r+2} = \frac{r-2}{r+1} a_r \quad \text{(Recurrence rel}^{n}\text{)}$$

$r=0 \rightsquigarrow a_2 = -2a_0$

$r=1 \rightsquigarrow a_3 = \frac{-1}{2} a_1$

$r=2 \rightsquigarrow a_4 = 0$

$r=3 \rightsquigarrow a_5 = \frac{1}{4} a_3 = -\frac{1}{8} a_1$

$r=4 \rightsquigarrow a_6 = \frac{2}{5} a_4 = 0$

$r=5 \rightsquigarrow a_7 = \frac{3}{6} a_5 = \frac{1}{2} a_5 = -\frac{1}{16} a_1$

$r=6 \rightsquigarrow a_8 = \frac{4}{7} a_6 = 0$

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \cdots$$

$$y = a_0 + a_1 x - 2a_0 x^2 - \frac{1}{2} a_1 x^3 + 0 - \frac{1}{8} x^5 + 0 - \frac{1}{16} x^7 + 0$$

$$\boxed{y = a_0 [1 - 2x^2] + a_1 \left[x - \frac{1}{2}x^3 - \frac{1}{8}x^5 - \frac{1}{16}x^7 - \cdots\right]}$$

Q4. Solve the PDE using charpit's method.

$$pxy + pq + qy = yz$$

$$f(x,y,z,p,q) = pxy + pq + qy - yz = 0$$

$\frac{\partial f}{\partial x} = py$; $\frac{\partial f}{\partial y} = px + q$; $\frac{\partial f}{\partial z} = -y$;

$\frac{\partial f}{\partial p} = xy + q$; $\frac{\partial f}{\partial q} = p + y$

Charpit's subsidiary eq^{ns}.

$$\frac{dp}{\frac{\partial f}{\partial x} + p\frac{\partial f}{\partial z}} = \frac{dq}{\frac{\partial f}{\partial y} + q\frac{\partial f}{\partial z}} = \frac{dz}{-p\frac{\partial f}{\partial p} - q\frac{\partial f}{\partial q}} = \frac{dx}{-\partial f/\partial p} = \frac{dy}{-\partial f/\partial q} = \lambda$$

$$\frac{dp}{py-py} = \frac{dq}{px+q-qy} = \frac{dz}{-pxy-2pq-qy} = \frac{dx}{-xy-q} = \frac{dz}{-p-y} = \lambda$$

$$\frac{dp}{0} = \lambda \rightsquigarrow \int dp = \int 0 \to p = a \text{, say.}$$

$$axy + aq + qy - yz = 0$$
$$q(a+y) = yz - axy$$
$$q = \frac{yz - axy}{a+y}$$

$$dz = p\,dx + q\,dy$$
$$dz = a\,dx + \frac{yz - axy}{a+y}\,dy$$
$$dz - a\,dx = \frac{y(z-ax)}{a+y}\,dy$$
$$\int \frac{dz - a\,dx}{z - ax} = \int \frac{y}{a+y}\,dy \qquad a+y = u \rightsquigarrow dy = du$$
$$\log(z-ax) = \int \frac{u-a}{u}\,du$$
$$\log(z-ax) = \int 1 - \frac{a}{u}\,du$$
$$\log(z-ay) = u - a\log u + b$$
$$\boxed{\log(z-ay) = (a+y) + a\log(a+y) + b}$$

Q5. Find the recurrence relation and ~~indicial~~ indicial equation of the equation.

$$x(1-x)y'' + 4y' + 2y = 0$$

$x = 0 \to P(0) = 0(1-0) = 0 \rightsquigarrow$ singular pt

$$\lim_{x\to 0} \frac{\cancel{x}\,4}{\cancel{x}(1-x)} = 4 \text{ (finite)} \; ; \; \lim_{x\to 0} \frac{\cancel{x}\,x\,2}{\cancel{x}(1-x)} = 2 \text{ (finite)}$$

$\therefore$ regular singular pt.

∴ Frobenius method

$$y = x^m \rightarrow \frac{dy}{dx} = m x^{m-1} \rightarrow \frac{d^2y}{dx^2} = m(m-1)x^{m-2}$$

$$(x^2 - x)\, m(m-1)x^{m-2} + 4m x^{m-1} + 2x^m = 0$$

$$m(m-1)x^m - m(m-1)x^{m-1} + 4m x^{m-1} + 2x^m = 0$$

$$(m(m-1) + 2)x^m + (-m^2 + m + 4m)x^{m-1} = 0$$

indicial eqⁿ $\boxed{-m^2 + 5m = 0}$

$$m(m-5) = 0$$

$\boxed{m = 0 \;;\; m = 5}$ indicial roots

highest degree − lowest degree $= \cancel{m}\, m - (m-1) = 1$

∴ Assume $y = \sum_{r=0}^{\infty} a_r x^{m+r}$

$$\frac{dy}{dx} = \sum a_r (m+r) x^{m+r-1}$$

$$\frac{d^2y}{dx^2} = \sum a_r (m+r)(m+r-1) x^{m+r-2}$$

$$(x^2 - x) \sum a_r (m+r)(m+r-1)x^{m+r-2} + 4\sum a_r (m+r) x^{m+r-1} + 2\sum a_r x^{m+r} = 0$$

$$\sum a_r (m+r)(m+r-1) x^{m+r} - \sum a_r (m+r)(m+r-1) x^{m+r-1}$$
$$+ 4\sum a_r (m+r) x^{m+r-1} + 2\sum a_r x^{m+r} = 0$$

$$\sum a_r [(m+r)(m+r-1) + 2] x^{m+r} + \sum a_r [-(m+r)(m+r-1) + 4(m+r)] x^{m+r-1} = 0$$

$\}\; r = r+1$

~~seriesza~~ " $+ \sum a_{r+1} [-(m+r+1)(m+r) + 4(m+r+1)] x^{m+r} = 0$

series = 0 → coefficient = 0

$$a_r[(m+r)(m+r-1) + 2] + a_{r+1}[-(m+r+1)(m+r) + 4(m+r+1)] = 0$$

$$a_r[(m+r)(m+r-1) + 2] + a_{r+1}[(m+r+1)[4 - m - r]] = 0$$

Recurrence relⁿ $\boxed{a_{r+1} = \frac{(m+r)(m+r-1) + 2\; a_r}{(m+r+1)(m+r-4)}}$

— x —

YASH VINAYVANSHI